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Children, Numeracy and Mathematics Recovery

Summary

We present the stories of three children in different countries who were struggling with early number. The governmental initiatives to raise standards are discussed but questioned as to whether they would meet the needs of children experiencing difficulty in the bottom 20 per cent of attainment. We show all three children have grown mathematically as a result of a detailed assessment leading to a teaching intervention which possessed clearly defined objectives and distinctive teaching approaches. Not only did each child attain, or surpass, the age-related norm but also as a result of the exercise teachers reported changes in their coverage of the curriculum and in their teaching style. The teachers also professed confidence to advise colleagues and to disseminate their learning and results.

We would like you to meet three young children. Judy is Australian and attends school in New South Wales, Denise lives in the North West of England and Michael is from South Carolina in the USA. Their classrooms are thousands of miles apart, in opposite hemispheres and in different time zones. Though they receive schooling under different educational administrations, Judy, Denise and Michael have several features in common. First, they are approximately the same age, between 6 and 7 years. Secondly, at the time of the interviews all three have been in school for almost a year and a half. They are regular attendees and well adjusted to school. However, they also share a problem. They are all experiencing difficulty with basic numeracy and are already falling behind the expected performance level of their year group.

Judy, Denise and Michael had been assessed in mathematical attainment and placed in the bottom 10 to 25 per cent bracket within their year group.

We know there is ample evidence (Aubrey, 1993; Wright, 1991b; 1994; Young-Loveridge, 1989; 1991) to show that there are significant differences in the numerical knowledge of children when they begin school and that these differences increase as they progress through school. In other words, children who are low attainers at the beginning of schooling tend to remain so, and the gap between them and the average to high attainers tends to increase. Thus the question arises: what will happen to these children if they do not receive additional support in numeracy other than that given by the class teacher as part of their normal duties? Further, since the vast majority of primary and elementary teachers are not specialist teachers of mathematics one might also ask whether the teachers are in a position to remediate the children's problems even if they know what these are. Since neither Judy, Denise nor Michael is disruptive, rather classified as 'cooperative', there is a strong possibility that their difficulties may be overlooked. Let us look at the actual attainment of each child and, though they are in different school systems, see if they experience common difficulties in numeracy lessons.

JUDY

Judy has completed one and a half year's schooling. Her teacher assessed her knowledge of the **number word** sequence, the ability to name **numerals**, and **strategies** for solving addition and subtraction involving two small collections of counters both of which were covered from view.

Judy showed she had a sound knowledge of the number word sequence in the range of one to ten. She could 'count' from one to ten and backwards from ten to one. She could also say the number before or after any number word in the range one to ten. However, she was not able to do this for the **numbers** in the teens or twenties. When numeral cards were shown to Judy she was able to identify the numerals in the range 1 to 20 but was not able to name many beyond 20.

Judy was given two verbal addition **tasks**, 5 + 4 and 8 + 3. First, she was presented with five red counters as a group, which were covered, and then four green counters which were placed under a separate screen. She was asked how many counters were there altogether. She successfully solved these tasks but it is revealing to consider the strategy she used in doing so. She invariably counted from one using her fingers or making pointing actions over the screened collections. Her strategy could be described as counting one of the screened collections from one and then continuing her count to include the other screened collection. 'One, two, three, four, five . . . six, seven, eight, nine . . . Nine!'

Judy demonstrates that she possesses one-to-one correspondence and has good counting skills but these are limited to the range one to ten. She identifies numerals up to twenty. She understands the operation of addition but not subtraction. It is important to note that her current number knowledge does not include number facts to ten. She uses a strategy of counting from one to solve additive tasks. Mathematically this is a low-level strategy. Given that she has difficulty with counting in the teens, her strategy is likely to fail her when larger quantities are introduced such as 9 + 6 for example.

DENISE

Denise was 7 years and 1 month at the time of assessment and had been in school for four terms. She could say the number words up to 27 but she had difficulty when saying the number that came after a number when it was above twelve. She also had a lack of fluency when crossing the decade number twenty. Her counting backwards skills were limited to saying the words from ten. When she was asked to start at fifteen and count backwards she said '15, 51, 52, 53'. She could say the number word that comes before a number but only if the number was less than ten. When asked to produce the number before a certain number between ten and twenty she always gave the number one more than, even when the task was repeated. For example, she would respond 'eighteen' when asked for the number before 17 and 'twenty-one' for 20.

Her numeral identification skills were limited to numbers less than 10. She said 'fourteen' for 47, 'fifty-five' for 15 and 'thirty-three' for 13. When Denise was given the same addition problem with the screened counters as Judy, that is, 5 + 4, her strategy was to guess 'seven'. The task was represented and this time she stated 'six'. Other than guessing Denise did not have a strategy for solving the addition tasks. The teacher reduced the complexity of the problem by showing her five counters which were then placed under a screen. Two more counters of a different colour were displayed alongside but this time they were not covered. She was asked, 'How many are there altogether?' The teacher indicated the total collection by a circular motion of the hand. Denise failed to answer three tasks, 5 + 2, 4 + 4 and 7 + 5, correctly. The level of the tasks was reduced even further by checking if Denise had one-to-one correspondence. First, a linear collection of 13

counters was presented and, secondly, one of 18 counters. Denise successfully demonstrated that she had good one-to-one correspondence in this range.

MICHAEL

Michael could count forwards up to 29 but halted at twenty. He could not say the number that came after a number within the range one to 29. He was not able to count backwards and as a consequence could not give the number which comes before a number. He was shown numeral cards to identify and was proficient in identifying the numbers up to 10. However, he had difficulties with the larger numbers especially the teens. For example, he said 'twenty' for 12 and 'fifty' for 15. He also failed to identify decade numbers.

When assessed on the additive tasks 5 + 4 and 8 + 3, he portrayed the same behaviours as Denise. He guessed. He was not able to solve tasks involving partially covered collections and he did not appear to be interested in trying to find a solution. He was successful, however, in demonstrating one-to-one correspondence with collections of 13 and then 18 items.

WHAT DO THE THREE HAVE IN COMMON?

We have presented the individual characteristics of the children but what attainments do they have in common? The number word and **numerals** knowledge and the strategies used for addition give us the clues. Collectively the three children have some proficiency in saying the forward number word sequence into the high twenties but all experience problems with teen and decade numbers. Judy and Denise can count backwards from ten but Michael is lost here. The lack of facility with number word sequences is also shown in the poor skills at saying the number which comes after or before a number word. Judy can identify numerals up to 20 but Denise and Michael's numeral identification skills are good only in the 1–10 range. Of the three children, Judy shows the most advanced strategy for the addition of two screened collections by the fact that she counts from one to arrive at the total. However, this is still a low-level strategy. Denise and Michael could not solve these problems, not even when they were posed using partially screened collections. But, they did have one-to-one correspondence. Overall, it is clear that they are performing at a low level of numeracy given the standard that is expected for their year group.

Our experience in assessing children in the three countries has led us to believe that Judy, Denise and Michael are not unusual in their level of mathematical knowledge. We are sure that teachers everywhere will readily be able to substitute the name of one, or more, of their children for the above.

The problem is not being neglected. The desire to raise standards of numeracy is a stated national education priority for many nations. Certainly Australia, the USA and England are currently undertaking systemic initiatives in early mathematics and numeracy.

In Australia, for example, the Commonwealth Government worked jointly with the states and territories to establish agreed National Numeracy Benchmarks for 8-year-olds. Commonwealth, state and territory ministers responsible for education have agreed the following literacy and numeracy goals: 'That every child leaving primary school should be numerate, and be able to read, write and spell at an appropriate level. That every child commencing school from 1998 will achieve a minimum acceptable literacy and numeracy standard within four years'. (*Numeracy = Everyone's Business*, p. 3, 1997).

In the past eight years many of the state and territory education systems in Australia, as well as the national education system in New Zealand, have developed and implemented major new

programmes in early numeracy. A detailed description of some of the most significant examples of these can be found in Bobis et al. (2005).

In the USA the Standards 2000 Project released the National Council of Teachers of Mathematics updated Standards entitled *Principles and Standards for School Mathematics*. The publication includes ten standards for children's learning which span pre-school through to grade 12. The Standards emphasize how learning should grow across the four grade bands – K–2, 3–5, 6–8 and 9–12 – and are seen as statements of criteria for excellence in school mathematics. Standard 1: Number and Operation, for example, states that mathematics instructional programmes should foster the development of number and operation sense so that all students:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- understand the meaning of operations and how they relate to one another.
- use computational tools and strategies fluently and estimate appropriately.

In the UK the government launched the National Numeracy Strategy which set out a Framework for Teaching Mathematics (DfEE, 1999a). The Framework provides a year-on-year list of key objectives that are expected to be attained by the 'great majority' of children.

Examination of the published documents for each country reveals a commonality in describing numerate children. The publications indicate the need for children to be confident and competent in working with numbers, money and measurement. Statements, listing the skills, knowledge and understanding to be attained, show that children should be able to calculate accurately and efficiently, both mentally and on paper, and have a sense of the size of a number and where it fits in the number system. They should have a knowledge of number facts and have strategies for solving problems. They should also be able to check the reasonableness of an answer and be able to explain their methods. They should recognize when it is appropriate to use calculators and have the skills to use them effectively.

We cannot disagree with these aims though clearly Judy, Denise and Michael are considerably behind in meeting the targets. Although the aims are laudable it is less clear how the improvement in attainment is to be achieved. How will the aims be translated into action to help Judy, Denise and Michael? The UK government's National Numeracy Strategy embarked on a rigorous training programme and published guidance on teaching mental calculation strategies (QCA, 1999a), exemplification of key learning objectives (QCA, 1999b), sample lessons (NNP, 1999) and lists of associated mathematical vocabulary (DfEE, 1999b). Let us now examine this initiative in greater detail.

The National Numeracy Strategy's Framework for Teaching Mathematics (DfEE, 1999a) provided a year-on-year list of key objectives which the 'great majority' of children were expected to attain. Each local education authority (LEA) has been given a target for improvement. The LEAs in turn have set targets for individual schools.

The National Numeracy Strategy has also provided schools with a mechanism for attaining the objectives based upon best practice from European and Pacific-rim countries. The mechanism is a structured three-part daily mathematics lesson. The sections are referred to as 'Introduction', 'Main Teaching Activity' and 'Plenary'.

A typical lesson commences with five to ten minutes' oral work and mental calculation with the whole class, which is conducted at a brisk pace. The aim here is to rehearse, sharpen and develop mental and oral skills that may be called upon in the lesson. This is followed by the main teaching activity lasting 30–40 minutes where the teacher works directly on new input with the whole class. They then have the opportunity to split into group work or individual and

paired work, thus providing consolidation and differentiation. The lesson concludes with a plenary session where the children explain their work and discuss the efficiency of different solution strategies. It is also the opportunity for the teacher to help the children to assess their developing skills against the targets they have been set and to record their progress, to make links with other work and to set homework.

Even if the target is reached by 75 per cent of the children, by implication 25 per cent are seen to be failing. Initially, prominence in the strategy was placed on whole-class introductions and summaries, with additional emphases on pace, mental agility and articulation of solutions and method. However, by 2004, despite some improvement being evident with underachieving pupils, greater recognition was being given to the fact that a significant number of children needed small-group intervention to accelerate progress and to allow the children to reach age-related expectations. Further some children would need individual provision to tackle fundamental errors and misconceptions that are preventing progress. Judy, Denise and Michael clearly fall into these categories and, if they do not receive help, will continue to experience failure and begin to feel that they are not part of the mathematics community in the classroom.

The National Numeracy standards indicate that children at Year 1 should have the following understanding of numbers and the number system, calculations and problem-solving skills:

1. Count reliably at least 20 objects.
2. Count-on and back in ones from any small number and in tens from and back to zero.
3. Read, write and order numbers from 0 to at least 20; understand and use the vocabulary for comparing and ordering these numbers.
4. Understand the operation of addition, and subtraction (as 'take away' and 'difference') and use the related vocabulary.
5. Know by heart all pairs of numbers with a total of ten.
6. Within the range 0 to 30, say the number that is one or ten more or less than any given number (QCA, 1999a, p. 6).

Clearly though Judy, Denise and Michael could meet 1 above, and partially meet 2, 3 and 6, they could not meet 5, and only Judy has a strategy for addition and this is a low-level one. Our concern is that teachers do not have sufficient skills to diagnose children's difficulties in numeracy and that in the desire to reduce the 'burden of assessment' greater emphasis is being placed on teacher assessment. We feel strongly that teachers need help in diagnosing errors and misconceptions and practical help and support firmly grounded in theory, to provide for the less able children in the early years. Such help was provided for Judy, Denise and Michael via the Mathematics Recovery Programme.

HELP FOR JUDY, DENISE AND MICHAEL VIA THE MATHEMATICS RECOVERY PROGRAMME

Judy, Denise and Michael, and their respective teachers, did receive help, support and training because each, in their own country, was selected to participate in the Mathematics Recovery Programme. A key feature of this programme is that it provides teachers with an extensive professional development course to improve their understanding, knowledge and skill in the teaching and assessment of early numeracy. This is achieved through participation in intensive, individualized teaching programmes for low-attaining children. The skills learned on the one-to-one basis can then be applied effectively to groups and whole classes of children.

Before explaining the short-term intervention programme in full detail let us show how it helped our three children by explaining the progress each made. We will also relate how knowledge of the assessment tasks, administrative style and the implementation of the intervention impacted on the teachers involved.

JUDY REVISITED

Following the Mathematics Recovery assessment Judy received individual teaching sessions of 25 minutes' duration, four times per week, for a total of seven weeks. This was a total of 11 hours and 40 minutes. The teaching activities were to enhance Judy's knowledge of the number word sequence and numerals, and to further develop her strategies for adding and subtracting. In selecting teaching activities the teacher's intention was to present problems that, on the one hand, were quite challenging but, on the other, were reasonably likely to be solved by the child. For the first three weeks of the teaching cycle Judy continued to adhere to her count-from-one strategy to solve addition and subtraction tasks.

During the fourth week of the teaching cycle Judy made a significant development in her strategies for adding and subtracting – she developed a strategy of **counting-on** rather than counting from one. Judy seemed to become aware of counting-on during her solution of the task of 20 + 3, which was presented verbally using two screened collections. She solved this task by counting from one to twenty and then continuing her counts from twenty-one to twenty-three, while keeping track of three counts. In an ensuing discussion with her teacher after having solving the task, Judy said 'twenty and three (pause), twenty-three!' Her statement seemed indicative of mental reflection on her solution to the task. From that point onward Judy routinely used counting-on to solve addition tasks and subtraction tasks. By the time of the fourth week of her teaching session Judy had also developed proficiency with the number word sequence in the range one to one hundred. Judy could now use counting-on to solve addition tasks presented verbally (that is, without use of written symbols), for example 87 + 5, and **missing addend** tasks presented verbally such as 42 + [ ] = 46.

By the end of the fourth week of the teaching cycle, it was apparent Judy had made significant progress in her **early number** knowledge, and there was little doubt that her progress was attributable to her participation in the MR teaching sessions. Also apparent was a very significant and positive change in Judy's general attitude to the MR teaching sessions. It was clear that Judy was keenly aware of her progress and success in Mathematics Recovery, and this awareness was accompanied by a very positive attitude towards participating in the teaching sessions. In her initial interview and in the early teaching sessions Judy tended to be somewhat quiet and withdrawn. Over the course of the teaching cycle there was a gradual and substantial change in her general disposition. By the time of the fourth week, for example, she seemed to relish undertaking tasks and solving problems. She would as much as challenge her teacher to present a difficult problem. Judy's MR teacher was fully aware of this change in Judy's attitude to the teaching sessions. Her classroom teacher observed that Judy was performing better in mathematics, exhibited an increased confidence in her approach to mathematics activities in the classroom and was more positive in her approach to all classroom activities.

DENISE REVISITED

Meanwhile Denise, in England, received four 25-minute lessons each week for a period of ten weeks. During that time she learned to count forwards and backwards from 112. She could say the number that came before, or after, a number in the range 1 to 100. She still had some hesitancy with certain decade numbers. She could identify all two-digit numerals and, like Judy, had developed a count-on strategy proudly exclaiming that she did not need to use her fingers. Denise's teacher had developed a game where a numeral card was turned over and a die was thrown to generate a subtraction problem. Denise turned over 14 and then threw a six.

Denise: That is a hard one, taking six away. I don't think I want to see that!
(Denise counts up to six.)
Denise: Took 14 away, 13 away, 12 away, 11 away, 10 away, 9 away, 8.
(As Denise was counting back she made a regular six pattern on the table.)

Denise had developed a **count-down-from** strategy which she used to explain how she answered the next problem, 18 – 5 = [ ].

Three lessons later Denise's teacher hid 14 bears in a cave. She told Denise there were 14 bears and asked her to close her eyes as she was going to remove some bears. She did this and informed Denise that there were only 11 bears left in the cave. How many had gone out? This can be summarized as a missing **subtrahend** 14 – [ ] = 11 and is a very challenging task.

Denise: Three!
Teacher: How did you do that?
Denise: I jumped in my head. Because 14 is not a jump we go 13, then 12, then 11.

Denise later solved 16 to 13, 20 to 16, 27 to 25 and 30 to 26. For 16 to 13 she showed double counting skills as she counted down to.

Denise: 15 that's one, 14 that's two, 13 that's three.

Denise had advanced her numerical skill and strategies for addition. She was now adept at counting-on and could **count-up-to** for missing addends. She had a count-down-from strategy for subtraction and was beginning to develop a **count-down-to** strategy though as yet she did not always choose the most appropriate strategy to solve a particular problem. Like Judy, she grew in confidence and enthusiasm. Most pleasing to see was the way in which she relished the challenge of mathematics and her ability to articulate her strategies.

MICHAEL REVISITED

Michael received a similar teaching programme to those of Judy and Denise. He showed tremendous gains in arithmetical strategies, in **forward** and **backward number word sequences**, and in **numeral identification**, all of which improved his self-confidence. When he was being assessed at the end of the programme he said, 'I'm going to do good thinking this time'. He was presented with a very difficult symbolic subtraction problem. The task on the card was 16 – 12. He read the problem out loud without hesitation.

Teacher: Do you have a way of figuring that out?
(Michael looked at the problem for about 15 seconds thinking hard. He looks up and says) Four.

Teacher: How did you figure that out?
(Michael shrugs his head and smiles broadly.)
Teacher: Your mouth was moving.
Michael: I took away 12 and there was four.
(As he was saying this he made a clenched fist with his left hand. His thumb and knuckles are facing him. He points to the little finger and says) Eleven was right there.

What is significant about this is that Michael was demonstrating a count-down-to strategy. He knew where 12 was and that if it had been 'subtract eleven' the answer would have been five. Michael also developed some 'known facts' which he used in addition and subtraction. Overall he showed considerable gains in strategies. His parents were delighted with the progress he had made and wished to share in it. Michael regularly took home a homework and communication book for numeracy. The lessons for the three children were routinely videotaped. Michael's parents watched some of the taught sessions to gain an understanding of the tasks and how they could reinforce the learning. Needless to say, Michael does not receive special education remediation and he has exceeded the standard expected of his age group.

CONCLUSION

It can be said that any child receiving individualized teaching should make progress. However, the progress of all three children, in different contexts and educational systems, is highly significant. They have achieved considerable gains and now possess advanced strategies for the solution of challenging addition and subtraction tasks. Also the Mathematics Recovery Programme can be seen to have had a very significant influence on the children's attitude to learning and their self-esteem. They no longer see themselves as failing.

Moreover, the programme has changed teachers' perceptions of what can be achieved and they have delighted in the ability to move children on from low-level to advanced strategies. During the course of their training in assessment and teaching they taught children individually. This was essentially to cut down control and management factors in order to concentrate on the teaching moves involved and the child's responses, a form of teaching experiment which can be described as microteaching. Teachers began to grow in confidence even before the teaching course input. The change came about when they began to work on the assessment schedules and associated frameworks. They reported that the aspects of the assessment schedules gave them a greater content knowledge of early numeracy together with greater insights into how children learn mathematically. They felt the training gave them clear direction in their teaching in that they knew exactly what the child could, and could not, do. They knew what objectives to set and how to achieve them. They began to put greater emphasis on less-used aspects of the curriculum such as backward number word sequences and number word before tasks. The assessment course not only changed their view of the curriculum but they reported how it began to impact on their teaching style. Because the emphasis in the assessment is on observation and interaction with the child, they began to pick up on the verbal and non-verbal clues the children give when solving tasks. They began to appreciate why children need time to answer and to check and reflect on their thinking. They told how they began to ask more probing questions to explore the children's responses. A significant effect was they professed a confidence and a willingness to help colleagues and to disseminate the new learning in their schools and districts.

We return to the children and their teachers in Chapter 10 to show how progress was made. In Chapter 2 we explain how you too may attain this knowledge and ability by presenting a learning framework for early numeracy.