CHAPTER 2. MODEL SPECIFICATION

The key difference between CFA and EFA lies in the opportunity to incorporate theoretical and substantive knowledge into the measurement analysis. With CFA, analysts must decide on which indicators measure which latent variables, whether measurement errors for the indicators are independent of each other or whether there are correlated errors, and, if there is more than one latent variable, which latent variables are related to each other. In addition, analysts may decide that some of the parameters in the model are fixed values or that sets of parameters are constrained to be equal. This process of defining the structure of the model and the model parameters is referred to as model specification. The specification of a measurement model is the first step in CFA.

In this chapter, we explore model specification for a variety of CFA measurement models. We begin with a simple model with a single latent variable to fix ideas and then introduce more complex models that involve multiple latent variables, indicators that are measures of multiple latent variables, method effects and correlated measurement errors, and indicators that are causes rather than effects of a latent variable.

To illustrate various measurement models, we draw on an example based on a set of measures for the concept of theological conservatism. In addition, for one of our illustrations, we also draw on two measures of religious attendance. These examples are inspired by a study using data from the 2008 General Social Survey (GSS) to examine theological conservatism among Christians in the United States (Hempel et al., 2012).

Table 2.1 provides the measures for theological conservatism as they appear in the GSS. These measures involve a mix of binary and ordinal indicators. The logic of CFA measurement model specification applies regardless of level of measurement of individual indicators. The level of measurement, however, is important to take into account when estimating model parameters. Chapter 3 provides an overview of estimators for continuous indicators, and Chapter 6 provides an overview of estimators for categorical indicators.

2.1 Forms of CFA Measurement Models

2.1.1 One Latent Variable

One of the simplest CFA measurement models is one that involves a single latent variable with a set of indicators and no correlations among the

Table 2.1: Indicator prompts and responses for theological conservatism indicators.

Prompt	Label
Would you say you have been "born again" or have had a "born again" experience—that is, a turning point in your life when you committed yourself to Christ? (Yes/No)	reborn
Have you ever tried to encourage someone to believe in Jesus Christ or to accept Jesus Christ as his or her savior? (Yes/No)	savesoul
Which of these statements comes closest to describing your feelings about the Bible? (1) The Bible is the actual word of God and is to be taken literally, word for word. (2) The Bible is the inspired word of God but not everything in it should be taken literally, (3) The Bible is an ancient book of fables, legends, history, and moral precepts recorded by men.	bible
Do you believe in Hell? (Yes, definitely/Yes, probably/No, probably not/No, definitely not)	hell

measurement errors. To give a few examples, such a model could be used to assess the psychometric properties of a new set of measures (e.g., measures of student attachment to a school), could be a first step in the development of a larger measurement model with multiple latent variables (e.g., examining the relationship between student attachment to a school and parent satisfaction with a school), or could form a baseline for assessing whether the measurement model holds across subpopulations (e.g., whether measures of student attachment to a school work similarly for boys and girls).

In this example and the following examples in this chapter, we adopt the variable labeling conventions commonly used in the presentation of CFAs with observed variables denoted by x, latent variables denoted by the Greek letter ξ (xi), and measurement errors denoted by the Greek letter δ (delta). In the broader structural equation modeling (SEM) context, these labeling conventions are reserved for exogenous latent variables (i.e., variables external to the processes modeled). Endogenous latent variables, or those embedded in the processes being modeled, come with a different set of labels: y for

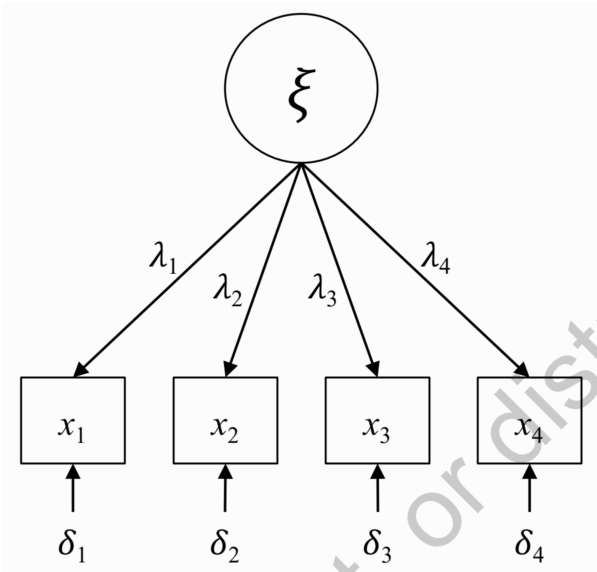

Figure 2.1: Confirmatory factor analysis measurement model with one latent variable and four indicators.

the indicators, the Greek letter η (eta) for the latent variables, and the Greek letter ε (epsilon) for the measurement errors.

Figure 2.1 depicts a measurement model with a single latent variable (ξ) and four indicators of the latent variable. Each of the four indicators (x_1 to x_4) also has a measurement error (δs), and there are no correlations among the measurement errors as evidenced by the lack of two-headed arrows connecting any of the measurement errors. Finally, the figure also includes the factor loadings, labeled with the Greek letter λ (lambda), that capture the effects of the latent variable on the indicators. This measurement model encodes the following theoretical or substantive expectations for the four indicators: (1) the four indicators are all measures of a single underlying latent variable, (2) the indicators are imperfect measures of the latent variable in that they include measurement error, and (3) the latent variable accounts for all the shared variance across the four indicators.

The measurement model illustrated in Figure 2.1 can also be represented as a system of equations as

$$\begin{aligned} x_{1i} &= \alpha_1 + \lambda_1 \xi_i + \delta_{1i} \\ x_{2i} &= \alpha_2 + \lambda_2 \xi_i + \delta_{2i} \\ x_{3i} &= \alpha_3 + \lambda_3 \xi_i + \delta_{3i} \\ x_{4i} &= \alpha_4 + \lambda_4 \xi_i + \delta_{4i}, \end{aligned} \tag{2.1}$$

where i indexes cases, αs are intercepts, λs are regression slopes or loadings, and δs are residual error terms. From this system of equations, we can see that the equation for each indicator looks much like a standard linear regression model. We have an intercept for each indicator, a regression coefficient (factor loading) giving the effect of the latent variable on the indicator, and an error term. As in a regression model, we typically assume that the mean of the errors is 0 and so the parameter of interest is the variance of the error. In contrast to a standard regression model, our predictor or independent variable is latent or unobserved. In CFA measurement models, we are often interested in the mean and variance of the latent variable(s). In total, the parameters for this model include four intercepts, four factor loadings, four measurement error variances, a mean for the latent variable, and a variance for the latent variable. As we will discuss in Chapter 3, however, we will need to fix some of these parameters to 0 or 1 in order to identify the model.

Figure 2.2 illustrates a measurement model for theological conservatism that has this form. We have the four measures of theological conservatism, *reborn*, *savesoul*, *bible*, and *hell*, represented as indicators of the latent variable for theological conservatism. In this figure, we substitute 1 in place of the first factor loading to scale the latent variable, an important component of model identification (see discussion in Chapter 3).

2.1.2 Two Latent Variables

In our first example, we examined the specification of a measurement model with only one latent variable. In many instances, however, researchers may wish to specify a measurement model with more than one latent variable. The need to include more than one latent variable can arise in several ways. First, it is possible that indicators of what are thought to be a single latent variable actually capture different dimensions of a concept and thus require more than one latent variable. For instance, one might have a set of indicators for religiosity that on reflection capture distinct domains of religiosity such as personal beliefs and engagement in religious services. In such a situation, a measurement model with latent variables for the different domains

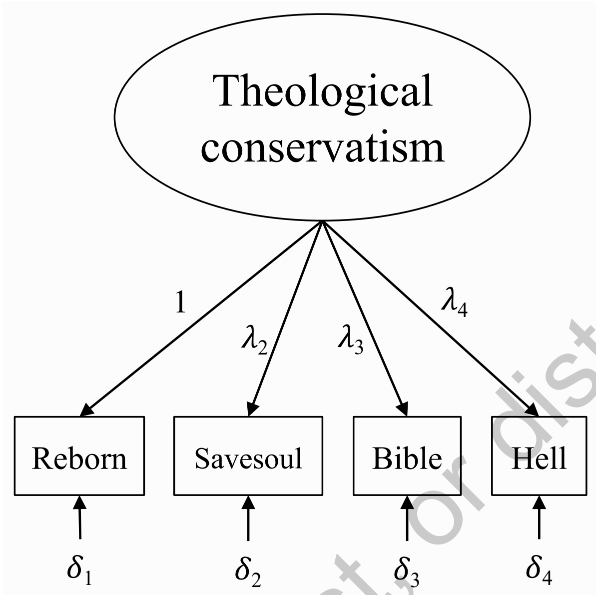

Figure 2.2: Confirmatory factor analysis measurement model for theological conservatism with four indicators.

of religiosity may perform better than a model with a single latent variable for overall religiosity. Second, the larger analytic context may involve examining the relationships between more than one latent variable. For example, a researcher may be interested in studying the relationship between student engagement and parent engagement in schools. After first developing measurement models for each separately, the researcher might combine the two into a single measurement model with both latent variables to estimate the association between student and parent engagement. Finally, in some cases latent variables may be used to capture method effects as in multitrait–multimethod (MTMM) models.

To extend the model from our first example, suppose that the analyst suspected that in fact two distinct latent variables were present that accounted for the shared variance across the four items. Figure 2.3 illustrates this model. In this model we see that x_1 and x_2 are indicators for the first latent variable and x_3 and x_4 are indicators for the second latent variable.

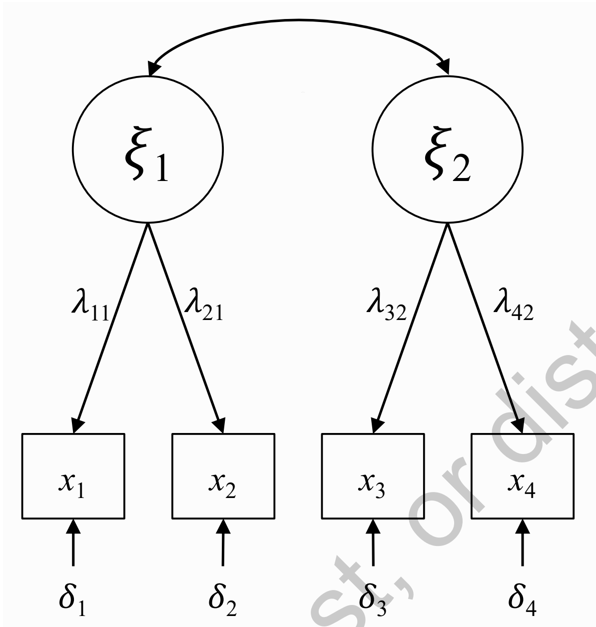

Figure 2.3: Confirmatory factor analysis measurement model with two latent variables and four indicators.

The system of equations for this measurement model are given by

$$\begin{aligned} x_{1i} &= \alpha_1 + \lambda_{11}\xi_{1i} + \delta_{1i} \\ x_{2i} &= \alpha_2 + \lambda_{21}\xi_{1i} + \delta_{2i} \\ x_{3i} &= \alpha_3 + \lambda_{32}\xi_{2i} + \delta_{3i} \\ x_{4i} &= \alpha_4 + \lambda_{42}\xi_{2i} + \delta_{4i}, \end{aligned} \tag{2.2}$$

where, by convention, we provide a second subscript on the factor loadings to indicate which latent variable has an effect on the indicator. For example, λ_{42} represents the parameter for the factor loading for the fourth indicator (where the 4 comes from) loading on the second latent variable (where the 2 comes from). The system of equations and the parameters are quite similar to the previous measurement model with one latent variable. We still have four intercepts, four factor loadings, and four measurement error variances. For this model, we add an additional mean and variance for the second latent variable and then also the covariance between the two latent variables.

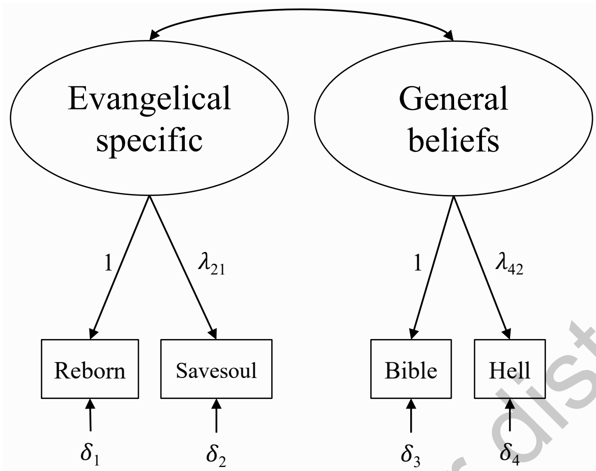

Figure 2.4: Two-dimensional confirmatory factor analysis measurement model for theological conservatism items.

Figure 2.4 illustrates an alternative measurement model for theological conservatism that instead posits that the concept consists of two dimensions: one that captures evangelical-specific beliefs and the other general beliefs regarding theological conservatism. In this case, *reborn* and *savesoul* might be best understood as indicators of evangelical-specific beliefs, while *bible* and *hell* might be best understood as indicators of more general Christian beliefs. Note that this measurement model structure implies that *reborn* and *savesoul* have a higher covariance than either has with *bible* and *hell*. To the extent this pattern among the covariances holds in the data, this measurement model will have a better fit with the data than the measurement model specifying only one latent variable. We discuss evaluating the overall measurement model fit and comparing the measurement models in Chapter 4. Some caution is warranted when working with CFAs that include only two indicators loading on a latent variable as special care must be taken to ensure such a model is identified (see Chapter 3).

2.1.3 *Factor Complexity Two*

It is possible that in some measurement situations an indicator may be influenced by more than one substantively meaningful latent variable. Generally, a measurement specialist creating an instrument prefers indicators that each measures only a single latent variable. In some traditions, such as Rasch

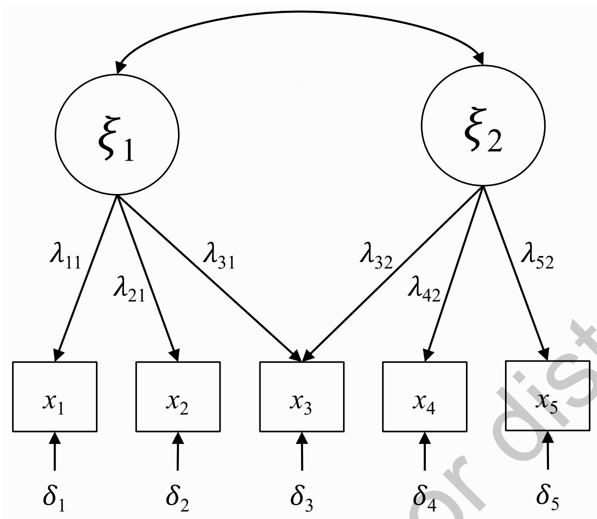

Figure 2.5: Factor complexity two confirmatory factor analysis measurement model.

modeling, it is even expected that each indicator will also have an identical loading as well as measuring only one factor. In this strict process, multidimensional items or items that measure more than one latent variable are removed (and usually considered to be poorly performing items for that reason). When performing measurement work on extant data, however, one does not always have the luxury of omitting items, and items that load on more than one latent variable can arise.

Measurement models in which one or more indicators load on two latent variables (or factors) are referred to as "factor complexity two" (FC2) models. Although rare in practice, this can be extended to multiple latent variables with factor complexity *n* models, where *n* is the greatest number of latent variables that influences a single indicator. Figure 2.5 illustrates an FC2 measurement model that involves two latent variables and five indicators. The first two indicators (x_1 and x_2) load on the first latent variable (ξ_1) and the last two indicators (x_4 and x_5) load on the second latent variable (ξ_2). The third indicator (x_3) loads on both latent variables. This set of effects for the third indicator is reflected in the system of equations for this measurement model given by

$$
\begin{aligned}
x_{1i} &= \alpha_1 + \lambda_{11}\xi_{1i} + \delta_{1i} \\
x_{2i} &= \alpha_2 + \lambda_{21}\xi_{1i} + \delta_{2i} \\
x_{3i} &= \alpha_3 + \lambda_{31}\xi_{1i} + \lambda_{32}\xi_{2i} + \delta_{3i} \\
x_{4i} &= \alpha_4 + \lambda_{42}\xi_{2i} + \delta_{4i} \\
x_{5i} &= \alpha_5 + \lambda_{52}\xi_{2i} + \delta_{5i},
\end{aligned}
\tag{2.3}
$$

where both latent variables appear in the equation for x_3. The factor loadings λ_{31} and λ_{32} capture the respective effects of the first and second latent variables on the indicator.

This model structure does not imply any covariances between the measurement errors for x_3 and the other indicators. The model structure also does not change our expectation that ξ_1 and ξ_2 covary. Other than the addition of one indicator, the sole difference between the model depicted in Figure 2.3 and the model depicted in Figure 2.5 is that one indicator is influenced by two latent variables rather than one.

Measurement models with factor complexity greater than one pose a couple of potential issues for analysts. First, as we will discuss in Chapter 3, it can be more challenging to specify models that are identified if numerous indicators load on more than one latent variable. Second, it is frequently the case in applied work that analysts make use of measurement instruments by creating a summed index of the items. Such summed indices are incapable of addressing the problem of factor complexity models, or within-item multidimensionality, as they are merely indices where each indicator is weighted equally, regardless of whether or not any one indicator is influenced by another latent variable. For this reason, when a practitioner expects their instruments to be used by analysts, they typically strive to remove the items that contribute to factor complexities greater than one from their instruments.

2.1.4 Correlated Measurement Errors

Suppose we return to our first measurement model with a single latent variable and four indicators of the latent variable. In specifying this model, we assume that all shared variance among the four indicators is due to their common dependence on a latent variable. There are a number of reasons an analyst might question this assumption. An analyst may suspect that method effects account for some of the covariance between the indicators. For instance, suppose an analyst is developing a measurement model for democracy based on four indicators and two of the indicators are ratings from the same international agency. Although some of the shared variance

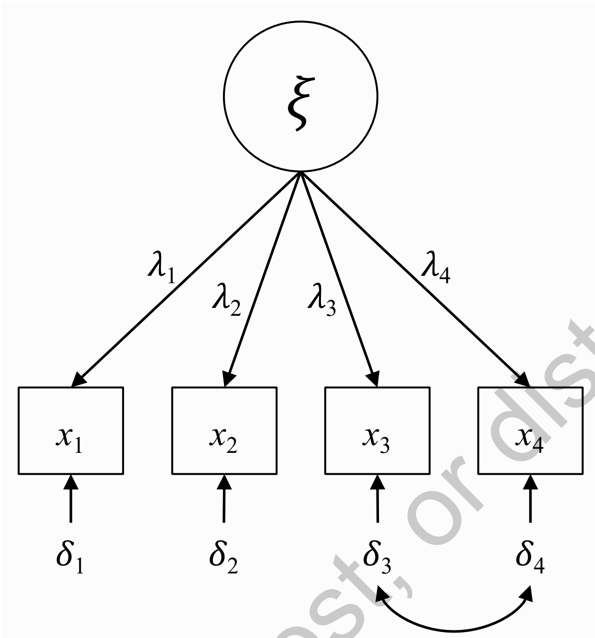

Figure 2.6: Confirmatory factor analysis measurement model with correlated measurement error.

in these two indicators reflects democracy, some shared variance may also reflect biases of the particular agency providing the ratings. Alternatively, similarly worded indicators from a survey, indicators with similar levels of reading difficulty, or indicators subject to social desirability bias, for instance, may all induce shared variance among sets of indicators and lead analysts to include correlations among measurement errors to account for the shared variance.

Figure 2.6 illustrates a measurement model that incorporates a correlation between the measurement errors for the third and fourth indicators as denoted by the two-headed arrow. The system of equations for this measurement model is the same as with Equation (2.1) and as such the model has the same set of parameters with one addition. With this model, we include the covariance between the measurement errors δ_3 and δ_4 as an additional parameter. A researcher might be tempted to allow all the measurement errors to be intercorrelated in order to capture any potential method effects or other forms of shared variance. Such a strategy, however, is not feasible

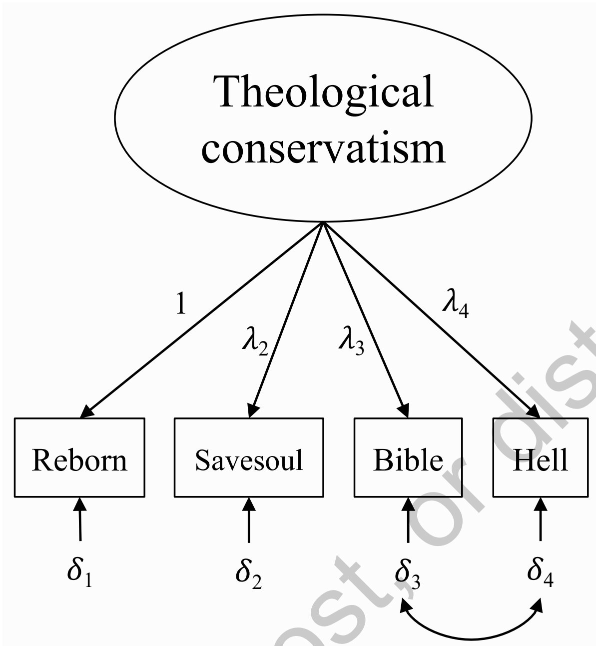

Figure 2.7: Confirmatory factor analysis measurement model for theological conservatism with correlated measurement error.

as each measurement error covariance requires a degree of freedom, and, as we will discuss in Chapter 3, researchers do not have sufficient degrees of freedom to spare for every possible measurement error covariance.

Figure 2.7 illustrates another possible measurement model for theological conservatism in which the errors for *bible* and *hell* are allowed to covary. As noted above, this model specification captures the possibility that the covariance between these indicators is not fully explained by latent theological conservatism. Such a possibility may be due to the fact that *bible* and *hell* are reverse coded or may be an artifact of the polytomous coding as compared with the dichotomous coding of *reborn* and *savesoul*. Alternately, such a possibility may reflect substantive differences in the indicators along the lines of our justification for the measurement model with two latent variable discussed above. In this case, rather than distinguishing two dimensions of theological conservatism, we might instead argue that all the indicators reflect theological conservatism but in addition *bible* and *hell* share additional covariance due to also reflecting general beliefs. This measurement model is closely related to the measurement model specifying two latent variables and, as we discuss in Chapter 4, it is not possible to distinguish empirically between the two measurement model specifications. As we discuss in Chapter 4, with a paucity of indicators (a situation many researchers will find themselves in, particularly when working with secondary data), researchers may find themselves at the limit of what the data can tell them about which model is best, and they must rely on theory in these cases.

2.1.5 Causal Indicators

The traditional CFA measurement model specifies indicators as caused by latent variables. Such indicators are commonly referred to as effect or reflective indicators. It is also possible to specify a measurement model in which indicators are causes of a latent variable rather than vice versa (Bollen and Bauldry, 2011; Bollen and Lennox, 1991). These indicators are referred to as causal or formative indicators. To give an example, socioeconomic status can be thought of as a latent variable measured by, for instance, education, income, and occupational status. If we think about these indicators, it seems unlikely that a change in socioeconomic status, the latent variable, would simultaneously lead to changes in all three indicators, as should be the case for effect indicators in which the latent variable is a cause of the indicators. Rather, a change in any of the indicators likely leads to a change in latent socioeconomic status. As such, a measurement model for socioeconomic status would be more correctly specified treating these three indicators as causal rather than effect indicators. Specifying such a model moves away from traditional CFAs and into the realm of SEM—for an extended discussion of these types of models, see Bollen and Bauldry (2011), and for a more complete treatment of the SEM framework, see Bollen (1989).

A measurement model may include a mix of causal and effect indicators as is illustrated in Figure 2.8. In this model, we have four observed measures of a single latent variable. Rather than treating all of the measures as effect indicators, as we have done in the previous models, we specify the first two measures as causal indicators and the third and fourth measures as effect indicators. The causal indicators have arrows pointing from the indicator to the latent variable to be consistent with the presumed direction of effects. In addition, the causal indicators are treated as exogenous, and thus they are allowed to be correlated and assumed to be free of measurement error (i.e., they do not have an error term associated with them). To emphasize that the causal indicators are still considered measures of the latent variable, we continue to use λ as the parameter label for the effect of the causal indicators on the latent variable. Finally, the latent variable, ξ, in this measurement

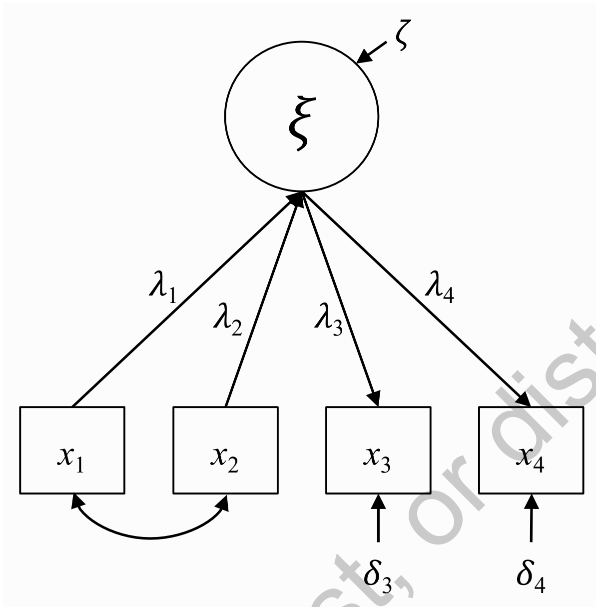

Figure 2.8: Measurement model with two causal indicators.

model is endogenous given that two of the indicators are predictors of it. This is reflected in having an error, labeled with the Greek letter ζ (zeta), pointing toward the latent variable in the figure.

The system of equations for the measurement depicted in Figure 2.8 is given as

$$\begin{aligned} \xi_i &= \alpha_\xi + \lambda_1 x_{1i} + \lambda_2 x_{2i} + \zeta \\ x_{3i} &= \alpha_3 + \lambda_{32}\xi_{2i} + \delta_{3i} \\ x_{4i} &= \alpha_4 + \lambda_{42}\xi_{2i} + \delta_{4i}, \end{aligned} \tag{2.4}$$

where the first equation in the system is for the latent variable. The parameters for this measurement model include three intercepts (one for the latent variable and two for the effect indicators), two factor loadings for the causal indicators, two factor loadings for the effect indicators, an error variance for the latent variable, two measurement error variances for the two effect indicators, and the correlation between the two causal indicators. As an endogenous variable, in this measurement model we have an intercept and an error variance for the latent variable as opposed to a mean and variance for the latent variable in a measurement model with all effect indicators. The speci-

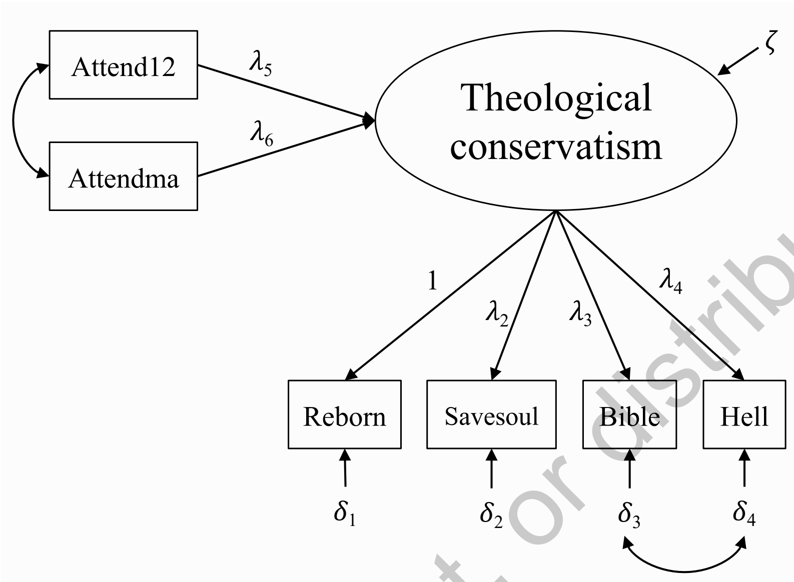

Figure 2.9: Measurement model for theological conservatism with two causal and four effect indicators.

fication of causal indicators and the resulting shift in parameters have implications for model identification, the topic of our next chapter.

If we turn back to the example of theological conservatism, we can introduce two additional observed variables as causal indicators. For instance, religious service attendance in adolescence (*attend12*) and mother's religious service attendance (*attendma*) are plausible indicators of theological conservatism. Given the temporal ordering of these indicators (i.e., measured based on adolescence as compared with adulthood for the other indicators), it seems more reasonable to treat them as causal rather than effect indicators. Figure 2.9 illustrates this measurement model that in addition maintains the correlated errors for *bible* and *hell*.

2.1.6 Observed Covariates

In general, with CFA we focus on measurement models in which all the observed variables are measures of latent variables. In some cases, however, it can be useful to incorporate the observed variables that are not indicators of a latent variable but instead are covariates (Bollen and Bauldry, 2011).

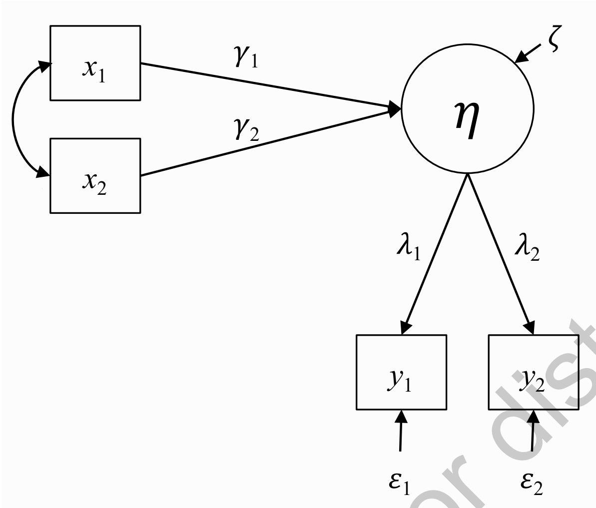

Figure 2.10: Model with two observed covariates.

This arises in the context of testing measurement invariance (see Chapter 5) and also when CFA is embedded within a larger analytic project that involves examining observed correlates, predictors, or outcomes of latent variables.

Figure 2.10 provides an example of such a model, in this case a multiple-indicator multiple-cause (MIMIC) model. In this model, we have two covariates, x_1 and x_2, that predict the latent variable. For instance, the latent variable might be religiosity as measured by two indicators and the predictors might be sociodemographic factors, such as age and gender. In this model, we label the effects of x_1 and x_2 on the latent variable using the Greek letter γ. In addition, we have switched to the endogenous latent variable notation with the Greek letter η for the latent variable, ys for the indicators of the latent variable, and εs for the measurement errors.

Attentive readers will note that despite the different notation and layout, this measurement model is identical to the model with causal indicators. The position of the covariates to the left of the latent variable rather than beneath it emphasizes that they are interpreted as predictors rather than causal indicators of the latent variable. The system of equations and the set of parameters for this model and the previous model are also identical. The identical model specifications for the previous model with causal indicators and the MIMIC model in this example highlight the critical role of theory in model specification and, ultimately, the interpretation of the estimates of CFA.

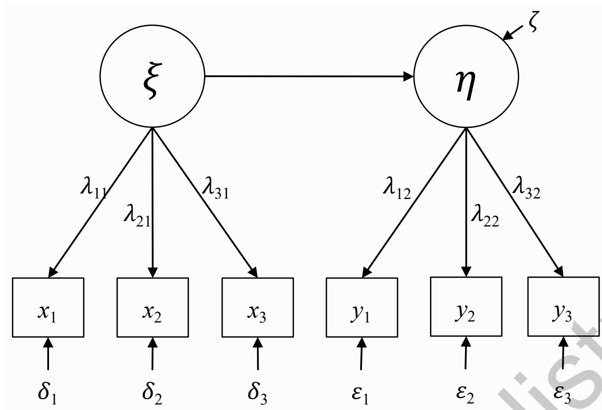

Figure 2.11: Structural equation model involving two latent variables.

2.1.7 Causal Relationship Between Latent Variables

In many cases, CFA is the first step in a larger analysis examining the relationships among a set of latent variables. If analysts are simply interested in the correlations between two or more latent variables, then the CFA framework is sufficient. If analysts, however, are interested in examining the effects of latent variables on other latent variables, then the broader SEM framework is needed. Given that such relationships are often of interest to analysts, we provide one example that embeds CFA in the more general SEM framework.

Figure 2.11 illustrates a structural equation model that involves two latent variables and a direct effect from one latent variable to the other. In this case, we have an exogenous latent variable ξ measured by three indicators x_1 to x_3 and an endogenous latent variable η also measured by three indicators y_1 to y_3. The latent exogenous variable has a direct effect on the latent endogenous variable given by γ. Previous steps in the analysis might have involved developing the measurement models for each latent variable separately and then combining them into a single measurement model with a correlation between the two latent variables.

The system of equations for the structural equation model depicted in Figure 2.11 is given by

$$\begin{aligned}
\eta_i &= \alpha_\eta + \gamma_1 \xi_i + \zeta \\
x_{1i} &= \alpha_{x1} + \lambda_{11} \xi_i + \delta_{1i} \\
x_{2i} &= \alpha_{x2} + \lambda_{21} \xi_i + \delta_{2i} \\
x_{3i} &= \alpha_{x3} + \lambda_{31} \xi_i + \delta_{3i} \\
y_{1i} &= \alpha_{y1} + \lambda_{12} \eta_i + \delta_{1i} \\
y_{2i} &= \alpha_{y2} + \lambda_{22} \eta_i + \delta_{2i} \\
y_{3i} &= \alpha_{y3} + \lambda_{32} \eta_i + \delta_{3i},
\end{aligned} \tag{2.5}$$

where the first equation captures the relationship between the two latent variables and the remaining equations are the measurement component of the model for the two latent variables. We can see that in addition to the standard parameters from the measurement model (i.e., the indicator intercepts, factor loadings, and measurement error variances), we also have the regression coefficient for the effect of ξ on η the intercept for η and the variance of the error for η. It is worth noting that the number of parameters from this model matches the number of parameters from a model in which the two latent variables are correlated with each other and therefore there is no statistical basis for adjudicating between the structural equation model depicted in Figure 2.11 and an analogous CFA measurement model with correlated latent variables.

2.2 Conclusion

In this chapter, we introduced many of the common forms CFA measurement models can take alongside a series of examples that illustrate some of the complexity involved in specific applications. As we noted above, the first step in any CFA involves determining the concept or set of concepts to be represented as latent variables and deciding how the various observed indicators relate to the latent variables. In some contexts, this is a straightforward process as a set of indicators or measures are developed or designed to measure a specific latent variable (e.g., items to measure depressive symptoms). In other contexts, however, analysts may be working with indicators that were not initially designed to measure a specific latent variable, in which case more thought needs to given to defining a latent variable and the indicators of it. Once an analyst settles on one or more latent variables and the candidate indicators, the second step in any CFA involves specifying a measurement model along the lines we have outlined throughout this chapter.

2.3 Further Reading

Although we have discussed the most common forms of measurement models, there are a number of additional forms used in special cases. For readers interested in higher order measurement models in which latent variables are measured by other latent variables, see Brown (2015) for a discussion. For readers specifically interested in examining construct validity, an MTMM measurement model can be valuable Campbell and Fiske (1959). For an extended discussions of the distinctions between causal indicators, covariates, and composite measures, see Bollen and Bauldry (2011). For a classic exemplar of a four-indicator model, see Bollen (1982). An exemplar demonstrating model specification with correlated errors may be found in Roos (2014).