WHAT YOUR COLLEAGUES ARE SAYING . . .

One of the saddest comments we often hear is "I was never any good at math." People blame themselves or the math. Rarely do they blame the mismatch between their cognitive and emotional needs and how they were being taught. In this engaging book, Lidia Gonzalez shines a light on the cultural, curricular, and classroom realities that are the real culprits.

—Steve Leinwand
American Institutes for Research
Washington, DC

Insight into why we need to change the narrative, "I'm bad at math!" So many moments of "*Yes*, you hit the nail on the head!" Authentic stories and compelling evidence reveal how our society continues to perpetuate this harmful myth. There are abundant resources to help stakeholders dismantle systemic barriers that persist in math and math education and reflection questions for education professionals. Awesome work!

—Shelly M. Jones
Professor, Mathematics Education,
Central Connecticut State University
Hamden, CT

Bad at Math? creates the space to unpack people's dispositions about mathematics. Many people dislike how mathematics is used to position them as either competent or incompetent. This book provides the content and context for people to unpack mathematics as the tool that helps us critique and understand the world.

—Robert Q. Berry III
Dean and Professor, College of Education,
University of Arizona
Tucson, AZ

This book was a pleasure to read and reread! Though the main discussion is mathematics, it should be a must-read for all preschool through higher education professionals. It's well written, and deeply rooted research tells the story. The long overdue, honest discussion is chock full of inclusive history and timely strategies positioning us to move forward and do better!

—Michele R. Dean
Field Placement Director and Lecturer, Graduate School of Education,
California Lutheran University
Thousand Oaks, CA

What a powerful and thought-provoking book! Gonzalez does a masterful job of addressing what is wrong with mathematics education currently and what can be done to make mathematics more accessible for more students, particularly those who are marginalized. Through these changes, we can help make it less socially acceptable for people to say they are bad at math.

—Kevin J. Dykema
President, National Council of Teachers of Mathematics (2022–2024);
Eighth-Grade Mathematics Teacher, Mattawan Middle School
Mattawan, MI

This book truly breaks down cultural norms to build up a powerful vision of mathematics for everyone. The engaging and thought-provoking discussions are paired with rich examples and resources that collectively create a powerful message to help us change the way math is perceived and achieved in schools. An important book for all education stakeholders!

—Jennifer Bay-Williams
Professor, University of Louisville
Pewee Valley, KY

Bad at Math?

To my daughter Sofia—you are the light of my life—
and to my father whose light, unfortunately, went out too soon.

Bad at Math?

Dismantling Harmful Beliefs That Hinder Equitable Mathematics Education

Lidia Gonzalez

CORWIN Mathematics

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You can access a Book Study Guide for *Bad at Math?* at **resources.corwin.com/badatmath**

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BAD AT MATH? AN INTRODUCTION

The most common response I get when I tell someone that I teach math for a living is that they were never good at math. There are variations to this answer, of course, but the essential point is that the individual I am speaking to is *bad at math*. Have you heard this comment or even said it yourself at times? Personally, I have gotten this response throughout my entire adult life both from those who have had little formal schooling and from those who have terminal degrees in their fields. As a high school teacher, when speaking to parents about the difficulties their child was having in mathematics, I was frequently surprised by their admission that they too had trouble with math. Now, as a college professor, I'm told by students and other faculty alike that they are *bad at math*. Among my colleagues, this is striking and perhaps unexpected. We are people who have devoted our lives to the pursuit of knowledge, yet, even among these learned individuals, I hear the *bad at math* comment frequently. Let me make this clear: Individuals who possess a PhD, who have published articles and books, who create new knowledge through research, and who are, by all the traditional measures, smart are comfortable saying that they are not good at mathematics. And they are not the only ones.

During the third quarter of Super Bowl LV, the Tampa Bay Buccaneers led the Kansas City Chiefs 24 to 9. The Buccaneers had possession of the ball and seemed poised to score again. One of the CBS announcers noted that he felt the Chiefs could come back at this point—being down by 15 points—but said that if Tampa Bay scored a touchdown or, perhaps, even two, this would no longer be true. Then he fumbled through the calculation of how many points behind the Chiefs would be should each of these scenarios occur. At one point he said, "I have trouble with the nines," referring to the fact that the calculations involved subtracting 9 points from the Buccaneers' total in each case. There were an estimated 96.4 million people viewing Super Bowl LV on CBS as he said this. The fact that so many people across all walks of life are comfortable publicly admitting they are *bad at math* doesn't sit right with me. Some might feel shame admitting that they cannot read, yet when it comes to mathematics, people openly admit their inability to do math seemingly without shame or hesitation.

As a mathematics educator, I would love for all individuals to appreciate and understand mathematics the way that I do. The feeling I get when I finally understand a problem I have been struggling with for weeks is quite addictive and one of the reasons why I decided to study math in the first place. I believe wholeheartedly that many more people can grow to both love and excel at mathematics than currently do. They too can experience the rush that comes from finally seeing a way through a problem that one has been working on. However, for this to be the case, the way in which we conceptualize and teach mathematics in our society needs to fundamentally change. So, too, must our acceptance of the socially constructed belief that it is permissible for many among us to be bad at the subject. The implications of society's acceptance of this belief and related beliefs are vast; they have a profound effect on mathematics education and, as a result, on our society as a whole.

Throughout the 20-plus years during which I have been teaching mathematics, many educational reforms have come and gone. The stated goals of adopting them have included increasing the number of students who excel in mathematics. These reforms have not truly been successful for myriad reasons. One is that the reforms have lived alongside the belief that to be bad at math is normal and to be expected. That is, there exists a crosscurrent that erodes efforts made to engage all students in the successful study of mathematics. Reforms, however well intentioned they may be, fall short when society fails to fully believe that they can be successful in a way that ensures that all (or even most) students excel in mathematics.

Reforms, however well intentioned they may be, fall short when society fails to fully believe that they can be successful in a way that ensures that all (or even most) students excel in mathematics.

In their book, *The Stories We Tell: Math, Race, Bias and Opportunity*, Faulkner et al. (2019) talk about *belief stories* and their ability to influence our decisions even in the face of data that contradict the story itself. For example, if we believe that Black and Latinx students struggle with mathematics more than white students do, whether we admit this to ourselves or the belief operates more subconsciously, we will be less likely to refer Black and Latinx students to accelerated mathematics programs, even if the students we are selecting for such programs have similar academic records. Faulkner et al.'s work highlights the fact that blind referrals made without knowledge of the students' gender, race, and ethnicity lead to a greater number of Black and Latinx students being referred to advanced programs.

BELIEFS AROUND MATHEMATICS AND MATHEMATICS EDUCATION

The acceptance of failure in mathematics, just like all belief stories, permeates our society. It is perpetuated in the media and cemented in our popular culture. More troubling, however, is that this acceptance finds its way inside our classrooms, boardrooms, and government agencies. It impacts decisions around pedagogy, policy, and practice and affects the lives of those who must live with the consequences of such decisions. A further result of society's belief that it is okay to be *bad at math* is a narrowing of the conversation such that blame for failure is placed squarely on the individual. ***I*** *am bad at math*. Given this, the way to resolve the problem is for me—the individual—to receive tutoring, participate in a support program, dedicate more time to doing mathematics, or any number of other interventions. In framing difficulties in mathematics this way, we neglect the broader issues that impact mathematics education. We fail to consider the impact of a system of public education that is deprived of resources, one that disenfranchises students from marginalized communities, and one that often fails to support, value, and treat teachers like the professionals they are. We further neglect to push back against curricula that center algebra above all other branches of mathematics, textbooks that do not adequately reflect our students or value their lived experiences, and standardized exams that fail to adequately capture our students' abilities.

Similarly, we have, as a society, constructed other beliefs around mathematics and mathematics education that if not dismantled are harmful to the students we serve and the larger society of which we are a part. Here are some of those other dangerous beliefs:

1. Mathematics is all about numbers and equations.
2. Mathematics is about getting to the one correct—the *only* correct—answer.
3. Someone who does mathematics is smart, and part of what it means to be smart is to be able to do computations quickly in one's head without the need for aids or research.

4. There exist a small number of *math people* for whom mathematics comes naturally.
5. The educational system is somehow irreparably broken.
6. There exist achievement gaps in mathematics.
7. It is not important to attend to identity when teaching mathematics.
8. Mathematics is neutral and its teaching apolitical.

Each of these commonly held beliefs impacts the teaching and learning of mathematics. Further, they frame the discussion around mathematics education; they define teacher preparation programs; they are reflected in teacher licensing requirements; they inform the development of policies, funding, and curricula; and in the end, they have a broad and lasting effect on the teaching and learning of mathematics and the students we aim to serve. We need to acknowledge that the way we frame mathematics and mathematics education also forces upon us ways of responding, engaging, and reforming the discipline. Thus, efforts at meaningful and sustained change for the better require us to attend to these constructs.

Additionally, we cannot separate our discussion of the beliefs that frame mathematics and mathematics education from the society in which this education system is embedded. Dr. Jean Anyon (1997), an educational researcher who explored the inequities around schooling in U.S. society, put it very clearly when she said, "attempting to fix inner city schools without fixing the city in which they are embedded is like trying to clean the air on one side of a screen door" (p. 168). At this point in history, we can no longer deny that our society is built upon institutionalized racism, which fundamentally affects our system of schooling and thus the teaching and learning of mathematics. Additional forms of oppression such as sexism, ageism, ableism, heterosexism, and classism have impacted and continue to impact the development of the systems of public education that exist in many places in the world today—especially in the United States and Canada. These forms of oppression play a pivotal role in the lives of the students and families served by the school systems therein as well as the lives of the faculty, administrators, and staff who work in them. Any attempt to improve mathematics education must acknowledge the fact that our educational systems—from classroom interactions, to teacher preparation, to school funding, to curriculum—exist within societies that are rife with inequality and in which power and privilege play a prominent role.

> **Any attempt to improve mathematics education must acknowledge the fact that our educational system—from classroom interactions, to teacher preparation, to school funding, to curriculum—exist within a society that is rife with inequality and in which power and privilege play a prominent role.**

Therefore, any attempt to understand this system and dismantle the beliefs that drive the teaching and learning of mathematics must attend to these realities.

WHOM IS THIS BOOK FOR?

This book is written for all those with an interest in the teaching and learning of mathematics. Most especially, it is aimed at the teachers, administrators, and instructional leaders such as mathematics coaches, mentors, professional development providers, and teacher educators who work in schools today. You live the realities described herein and are uniquely positioned to lead efforts toward chipping away the harmful beliefs that currently exist—beliefs that, if not dismantled, severely limit efforts at improving the educational experiences of students with respect to mathematics. The ways in which you, as teachers, engage with students, parents, and the content you teach impact how mathematics is viewed and the beliefs that students and others cement around mathematics and mathematics education. As instructional leaders, you work with teachers to strengthen the ways that mathematics is taught. Your suggestions, your support, and the discussions you lead impact how these teachers conceive of mathematics, how they develop as educators, and what beliefs they pass on to their students. As administrators, you make decisions around curriculum, academic policies, budget, and hiring that impact the environment in which students, teachers, and instructional leaders work. It is this environment in which most people develop their beliefs about mathematics, and it is this environment where we can rewrite the story of what mathematics is, what it means to be good at it, and who can excel at it. You all, in your related and varied roles, have substantial influence on how mathematics will come to be seen and understood for generations to come.

HOW CAN THIS BOOK HELP?

The chapters that follow attempt to deconstruct commonly held beliefs about mathematics and/or mathematics education. Each chapter incorporates narrative and reflection into a discussion that highlights relevant research while paying particular attention to issues of power, privilege, and systems of oppression present in society. All of the chapters focus on the K–16 system in the United States and, to a lesser extent, Canada with a special emphasis on those schools that serve predominantly Black and Latinx students. They are also rooted in my experiences as an educator, a researcher, a student (the first in my family to go to college), a gay woman, the daughter of immigrants to the United States, and the parent of a school-aged child. Improving the mathematical experiences of those typically marginalized in mathematics is my passion and life's work. My hope, and the goal of this book, is that by critically examining the social constructs that frame mathematics and mathematics education, we can step outside the usual discourses, expand the conversation, and undertake authentic substantive changes toward equitable mathematics education.

This introduction is followed by 11 chapters. Chapter 1, *What Does It Mean to Be Good at Math?*, looks at the commonly held beliefs about what it means to be good at math and their implications. Chapter 2, *Beyond Numbers and Equations: What Is Mathematics?*, challenges traditionally held beliefs that center the definition of mathematics on numbers and algebra. In Chapter 3, *Mathematicians and Mathematicians in Training*, we examine commonly held beliefs about who mathematicians are and what they look like. We look at depictions of mathematicians in popular culture as places where stereotypes are reinforced. Chapter 4, *We Are All Math People*, confirms the existence of *math people* by redefining what that term means. In Chapter 5, *Identity in Mathematics Education*, we challenge the idea that mathematics education is such an objective discipline that it need not concern itself with issues of student identity. In Chapter 6, *School Mathematics*, we step back and uncover where many of our commonly held beliefs about mathematics and mathematics education originated: school. In Chapter 7, *Mathematics as Gatekeeper*, we examine the role that mathematics plays as a gatekeeper to future success. We look at mathematical testing and its role in our society. Chapters 8 and 9 move to system-level concerns surrounding education. In Chapter 8, *Achievement Gaps or Opportunity Gaps?*, we push back against commonly accepted narratives about achievement gaps between more affluent white students and their less mainstream peers. National discussions of the achievement gap are rampant within education, particularly as they relate to math and particularly in the wake of schools' reactions to the COVID-19 pandemic. In Chapter 9, *Is the School System Broken?*, we challenge commonly held beliefs about the purposes of schooling by considering the role that the educational system has as a reproducer of the inequality present in our social world. In Chapter 10, *Teaching Mathematics as a Political Act*, we challenge the commonly held idea that mathematics is neutral, objective, and apolitical. We consider the many ways that the teaching of mathematics is a political act in terms of what content is taught, whose stories are told, and how mathematics is contextualized. In the book's last chapter, *Where Do We Go From Here?*, we consider the power that you—as teachers, instructional leaders, and administrators—have to dismantle the harmful beliefs that currently exist around mathematics and mathematics education.

Each chapter ends with a series of questions for reflection aimed at teachers, instructional leaders, and administrators so that you can further engage with the ideas of the chapter. How can you, in your sphere of influence, take the ideas in the chapter and further them? How can you use your power, privilege, and position to act on the concepts therein to instill changes needed to strengthen mathematics education and best serve our most vulnerable students?

You will also find a Book Study Guide at resources.corwin.com/badatmath that is designed to help you and your colleagues work together as a community to digest the content in this book, try some of the activities together, and implement changes in your day-to-day practice.

I recognize that the topic of this book is complex, and the issues raised are beyond the influence of any one individual to fix. Only a collective effort will lead to needed change. I also recognize that I don't have all the answers—no one person does. For every facet of this topic, whole books could be written—and many have been! It would be impossible to cover it all in one single book, and I'm grateful to be able to stand upon and lean into the work of so many others striving collectively toward a more joyful and equitable mathematics learning experience for all. So, throughout this book, I try, where possible, to offer concrete ideas for small shifts you can start with, as well as additional resources such as books, websites, organizations, and podcasts to continue deepening your learning and practice.

HOW CAN YOU USE THIS BOOK?

This book should serve to ignite conversations at every level of education around what mathematics is, what it means to be good at mathematics, who is good at mathematics, and how the political and the mathematical are intertwined. It should serve as a springboard to talks about social justice and the ways mathematics education can promote it. It can be used in professional learning communities with in-service teachers as well as in courses for pre-service teachers. As the basis for professional development or a teacher book club read, this book has the potential to extend conversations already happening across pockets of North America. In the end, I call on you to build upon the work you already do to support equitable mathematics education and to join me in challenging the harmful beliefs that exist around mathematics and mathematics education so that there may come a day when no one is proud to announce they are *bad at math*.

A NOTE ON LANGUAGE

As Robin DiAngelo, best-selling author of *White Fragility* (2018), writes in her 2021 book *Nice Racism: How Progressive White People Perpetuate Racial Harm*, "Language is not neutral . . . the terms and phrases we use shape how we *perceive* or make meaning of what we observe" (p. xvii). As such, I acknowledge that the words I use in this text do not simply describe the realities discussed; they make meaning of them and reflect my own perceptions. Therefore, I feel it necessary before delving into the content of the book that I make clear some of my choices around language. When I am writing about racialized people, I use the current most recognized term for the group: *Black*, *Latinx*, and *Indigenous*. I do this even when the research I am citing differs in language (such as use of the term *Hispanic*) because I believe there is value in using terms that have evolved through time and are the current most recognized ones available. It is also in keeping with the current research in my field. I use the phrase *people of color* to describe all those who are nonwhite. I use the phrase *underrepresented groups in mathematics* in keeping with the definition of the National Science Foundation to describe those whose representation in mathematics degree programs and math-related careers is lower than their representation in the U.S. population (National Center for Science and Engineering Statistics, 2021). I use the phrase *students typically marginalized in mathematics* to describe not only those who are underrepresented but all who have been pushed to the margins of this discipline.

I often juxtapose the educational opportunities typically afforded to white students in contrast to those afforded to people of color, creating a binary dynamic that I understand fails to capture the many experiences and cultures present in both groups. This is, in part, due to the constraints of the research cited, of language in general, and of my own inability to find a way to address these issues while attending more adequately to the diversity within each group. I understand that collapsing peoples into these groups erases the nuances that exist within them, but hope that for the purposes herein these broad categories serve as useful and aligned with the current research literature. I capitalize *Black*, *Latinx*, and *Indigenous*. I once capitalized *white*, as it too is a racial category, but later learned that this is a practice among white supremacists. Not wanting to emulate such people, I no longer capitalize *white*.

When using terms like *men*, *women*, *girls*, and *boys*, I refer to all who identify with these terms—to people's primary identities. I acknowledge that some do not identify with either gender or do not do so consistently. I have avoided using *he* or *she* when gender is not known, choosing instead to use nongendered terms like *students*, *individuals*, and *people*, or the pronoun *they*. I could have added a third gender category but for brevity and flow did not. Some might be marginalized by this omission, and for that I do apologize.

I am hopeful that this clarifies some of the choices I have made with respect to language but acknowledge that no choice is perfect. I may be excluding some with my words while marginalizing others. Neither is intentional. I acknowledge too that I am part of the racialized system that exists in our society and that while I aim to be less racially oppressive, I too have been socialized into this system and play a role in the systemic racism that exists. I strive to be less ignorant on these matters, but my attempts are imperfect as well.

CHAPTER 1

WHAT DOES IT MEAN TO BE GOOD AT MATH?

In this chapter we will:

- Discuss and challenge existing beliefs about what it means to be good at math and how those beliefs commonly lead to the *bad at math* trope.
- Explore the role of open-ended problems in mathematics education.
- Consider the role of productive struggle and a growth mindset.
- Explore research on the role of brilliance in mathematics.
- Reflect on how you can challenge traditional views of what it means to be good at math.

WHAT BELIEFS EXIST AROUND WHAT IT MEANS TO BE GOOD AT MATH?

Society has developed a set of beliefs around what it means to be good at mathematics. For one thing, many believe that being good at mathematics means being able to complete numeric computations in one's head both quickly and accurately. Therefore, speed and computational skills are essential. Further, it is often believed that being good at mathematics means one can work through mathematical problems using previously memorized algorithms and procedures without additional aids such as a textbook or calculator—that is, that research and other tools are not necessary for those who excel in the subject. Our beliefs about what it means to be good at mathematics impact how the subject is taught. These beliefs also affect those who might wish to pursue its study, and they drive how we engage with the subject. The belief that to be good at mathematics one must be able to carry out numeric computations with speed and accuracy is driven by several factors, including

1. the types of problems most commonly featured in mathematics classrooms,
2. a bias toward algebra and number sense in our curriculum, and
3. representations of mathematics and mathematicians in the media and elsewhere.

Let us focus now on the first factor—the types of problems used in mathematics classrooms—and return to the others in later chapters.

TEXTBOOK PROBLEMS VERSUS OPEN PROBLEMS

There is a bias in mathematics textbooks and classrooms toward problems that are narrow in scope and that practice previously taught procedures. Students are taught a set of steps to solve problems of a certain type and then given a set of problems that test whether they have learned that procedure. This means there is an overreliance on procedures and less opportunity for the creative thinking that comes with exploring a more open-ended problem or one where the steps have not been given ahead of time. Mathematicians do not work on problems that have a set of steps previously laid out for them, so there is a need for creativity and approaching problems in multiple ways. The kinds of problems we typically see in classroom settings might be good at getting students to practice a particular algorithm, but they do not invite the type of open-ended problem solving that requires creativity, multiple approaches, and thoughtful study over a prolonged period. As a result, students fall victim to the false idea that they should know how to do math problems quickly.

There is a difference between the exercises we generally see in textbooks and true thoughtful, open-ended problems. We better serve our students by providing opportunities for both. Open-ended problems, as contrasted with closed problems, are those for which there is more than one solution and for which there are multiple methods to a solution. As an example, rather than ask what the sum of 10 and 5 is, one can ask the following related open-ended question: *The sum of two numbers is 15. What could the two numbers be?* Similarly, one can ask a student to prove, however they want to, that 55 is greater than 37. Here the student can subtract, draw pictures, compare the tens, plot both numbers on a number line, and so on. Lastly, instead of asking students to graph a line whose equation is given, one can ask them to provide the graph and equation of multiple lines through the point (1,2). To have them play with open-ended problems breaks down students' misconceptions about the nature of mathematics and builds up their ability in the subject by requiring them to use mathematics in creative and flexible ways. It also allows for questions to arise that might have otherwise not come to the surface. Open-middle problems are those that, while they have one solution, can be solved using multiple methods. These, too, have a place in our classrooms as they allow for creativity in problem solving. The idea is to use problems that lead to true authentic student thinking. As articulated in his text *Building Thinking Classrooms in Mathematics*, Peter Liljedahl (2021) spent over a decade studying the conditions and behaviors that create the optimal conditions for student thinking in mathematics classrooms. He makes a distinction between problems that promote mimicking—re-creating the steps a teacher has just gone through with a very similar problem—and those that promote actual thinking about mathematics. In the second case, students use mathematics and build mathematics in creative ways. They think about mathematics by looking for patterns, exploring connections, or extending the knowledge they have of the subject in some way. It is problems that promote this kind of thinking that often get neglected, and it is these problems that we should strive to bring into our teaching to help foster students' confidence and competence.

Additionally, utilizing complex problems without overly scaffolding them conveys that mathematics problems take time and creativity. Consider exploring the question of how many squares there are on a chessboard. A quickly obtained response might be 64, but this only counts the 1×1 squares. The whole board is a square, and there are many other squares in the board as well. To explore problems such as this normalizes the idea that some mathematical problems cannot be solved quickly or by simply applying an algorithm or calculation. Believing math problems can be solved quickly is harmful to students who, when faced with a problem they do not know how to solve, internalize the belief that they must be *bad at math*.

Recently, a parent shared with me another thing that for younger students may lead to misconceptions about the nature of mathematics: the amount of space given to solve various problems in workbooks and on worksheets. She noted that her children are often frustrated that even for problems that ask for an explanation of one's work only a small amount of space is given to work within. This might contribute to students' belief that the work and the thinking that leads to the solution to a mathematical problem should be short in nature. It might even reinforce the idea that what matters is an answer and not necessarily all the trials, work, and thoughts that get them to that answer—even when explicitly asked for—given how little space is offered to reflect their thinking. Perhaps encouraging students to use a blank piece of paper with more space or a whiteboard or journal would grow the belief that some mathematical problems might take time and much work to solve. Have you considered the amount of space your students have to work out mathematics?

Additionally, if we are honest with students about the fact that understanding comes with time and that multiple unsuccessful attempts often precede the attempt where one solves the problem, we will be teaching students that what matters is persistence and that an inability to solve a problem quickly is not proof of their inadequacy, but part of what it means to engage with mathematics. Students who struggle—or, worse, those who are labeled as struggling learners—internalize the belief that they are not good at math. Yet, struggle benefits us. Brain research demonstrates that our minds grow when we make mistakes and struggle (Boaler, 2015a; Moser et al., 2011). It is through work on tough conceptual problems that allow for us to get both stuck and unstuck that we grow as learners.

Struggle in mathematics is to be expected, and those who are successful are not those who do not encounter such struggle, but those who are able to persist despite it.

Struggle in mathematics is to be expected, and those who are successful are not those who do not encounter such struggle, but those who are able to persist despite it.

To do mathematics is to be able to rethink one's approach, engage in research, try something new, regroup, and come at a problem in yet another way. It is a process known as *productive struggle*. Pasquale (2016) notes that when students struggle with a problem yet continue to make sense of it, they are engaging in productive struggle. Students who are most successful are those who can sit with the discomfort of not knowing and who are able to continue to work through problems regardless. Yet struggle is not what comes to mind when most people consider what it means to be good at mathematics.

I teach a mathematics course for future elementary school teachers in which I routinely have students work together in groups on an open-ended mathematics problem related to the topic for the day. These problems are not the typical, straightforward type that look the same as the one your professor just completed at the board but with different numbers. There is no set algorithm previously learned in class that can be applied to these problems. They require thought, creativity, and often multiple differing approaches before a group of students can see a way through to a solution. In the first few class sessions, students often take the problem and push it aside, claiming they don't know how to do this one or don't know the formula. They expect to be able to solve the problem quickly using a previously learned algorithm. If they cannot recall the algorithm, it must mean they cannot solve the problem.

I tell them that there is no formula and that they know enough to solve it, but that it requires them to *play* with the problem in much the same way that they would play with a puzzle, trying to fit a piece into various spots until they find where it belongs. Most are uncomfortable with the idea of playing in mathematics, though their comfort grows with each class meeting until they become comfortable at *play*. It takes time to begin to break down some of the ideas that students have about what it means to be good at mathematics, just as it will take time for society at large to rethink what it means to be good at mathematics. Nevertheless, this must occur.

There is a wonderful TED Talk, *Math Class Needs a Makeover*, by Dan Meyer (2010), in which he focuses on how we present mathematical content in mathematics classes in a way that provides for students all the information needed to solve a problem including visual representations, labels, symbolic language, and mathematical structure. We clean up the problems so much and create such small steps and substeps that students miss out on the opportunity to build mathematical understanding and to have mathematical conversations while considering authentic, rich problems. With respect to typical mass-produced textbooks, Meyer states, "What we are doing here is taking a compelling question, a compelling answer but we're paving a smooth, straight path from one to the other and congratulating our students for how well they can step over the small cracks in the way" (4:56). Instead, Meyer suggests teachers ask the shortest question possible and let the mathematics come out of the discussion that arises so that "math serves the conversation—the conversation doesn't serve math" (5:58). Instead of leading students through a discussion of slope, complete with the formula and notation needed, he models a situation where students are asked to consider what the steepest part of a ski lift is. By asking the shortest question, he begins a conversation that leads to a need for points, notation, symbols, and a way of defining steepness. The mathematics that comes out of that conversation is what you might expect to cover in a more typical lesson, except in this case the mathematics was developed, constructed, and built in conversation with students.

Meyer (2010) also challenges us to develop in our students the ability to be patient problem solvers. Typically, when students are asked to work on problems, they have a short amount of time to do so, yet complex mathematical problems take time to solve. There are starts and stops. You might try one approach and find the need to modify or abandon it altogether and try something new. You will often need to look up relevant research, see what others have tried, and note where the gaps are between what is already known and what needs to be known. There are problems that have taken centuries to solve and whose solutions have relied on the work of numerous mathematicians over the years. There are others that despite consistent and concentrated efforts over the course of decades, if not centuries, remain unsolved. In contrast to open-middle or open-ended problems, these are known simply as *open problems*.

One can, in fact, find lists of these open problems that have yet to be solved in mathematical journals or in books. There is a very well-known list of such problems put out by the Clay Mathematics Institute, a nonprofit organization dedicated to the mathematical research known as the millennium problems. The list of seven problems was released in 2000, and a $1 million prize is offered to anyone who solves one. The problems focus on a variety of subjects including number theory, algebraic geometry, and topology. An accessible explanation of the problems can be found at https://brilliant.org/wiki/millennium-prize-problems/. To date, only one of the millennium problems has been solved. It was known as the Poincaré conjecture. Proposed in 1904, it is about the topology of objects known as manifolds. It was solved by Grigori Perelman in 2006, a full 102 years after it was proposed. Interestingly, Perelman turned down the million-dollar prize and the Fields Medal, mathematics' highest award.

DEVELOPING RICH MATHEMATICS

Another difference between textbook exercises and open problems has to do with the mathematics that grows out of the work of solving these problems. It is common for new, valuable, rich mathematical ideas to be developed in the process of solving an open problem. While the solution to the problem might elude mathematicians for years, in working toward a solution, various techniques, methods, and mathematics are developed. Additionally, other related questions might arise for which solutions either are or are not able to be found. As an example, consider the Collatz conjecture, which is named after Lothar Collatz who put forth the idea in 1937. Begin with any counting number. If the number is even, you take half of it. If the number is odd, you multiply it by 3 and add 1. Now you have a new number and repeat the process. The chains for 5 and 7 are shown in Figure 1.1.

Figure 1.1 • *Collatz Conjecture Chains for 5 and 7*

Even? Divide by 2 Odd? Multiply by 3 and add 1	5 → 16 → 8 → 4 → 2 → 1 7 → 22 → 11 → 34 → 17 → 52 → 26 → 13 → 40 → 20 → 10 → 5 → 16 → 8 → 4 → 2 → 1

The conjecture states that if we start with a counting number, *any* counting number, and apply these steps, we will eventually get the number 1. You might be wondering what happens if we do not stop at 1. In this case we get the chain in Figure 1.2, which loops forever.

Figure 1.2 • *Collatz Conjecture Chain for 1*

1 → 4 → 2 → 1 → 4 → 2 → 1 → 4 → 2 . . .

No one knows for sure if every single number leads to a chain that eventually gets to 1, though many numbers have been checked. The fact that this conjecture remains open after over 80 years and yet is the subject of study has yielded interesting results and further questions for exploration. For example, questions have been asked about the length of the chains. The chains we created for 5 and 7 have six and seventeen numbers in them, respectively. The number 27, however, has 111 numbers in its chain. Which numbers yield very long chains, and which ones yield shorter chains? Chains of specific kinds of numbers have been investigated as well. For example, we can ask if there is anything interesting about the chains belonging to prime numbers. We can consider if there is anything interesting about the chains that belong to multiples of 7. The chain for 5 we created contains the number 8. We can ask what other chains contain the number 8. We can consider, too, questions around the frequency of even and odd numbers in certain chains. Rich problems such as the Collatz conjecture allow us not just to solve one problem, but also to discover, create, and see mathematical questions and avenues for further exploration. Even in this one example we can see how mathematics is alive with more and more questions unfolding before us. This example is given to highlight how an open problem, whether or not it is solved, leads to a plethora of new questions and avenues for exploration. These problems allow us to get lost in mathematics, following the questions as they arise in much the same way that one might get lost in a forest. In fact, getting lost in a mathematical forest is what Sunil Singh (2021) proposes to encourage mathematical wellness and inspire a love of the subject. Yet, while it might be nice to introduce a problem such as this so that students see the field as growing, students won't typically be working on problems that have gone unsolved for decades or more. They can, however, work on open-ended or open-middle problems with regularity, and doing so would benefit them.

Similar to how working on an open problem may yield new results and discoveries for a mathematician, students working on an open-ended problem (for which there may already exist a known solution) might discover mathematical patterns, use mathematics in ways that are new or innovative for them, and build their understanding of the field in a way not possible with a simple textbook exercise. Too few people have ever had the opportunity to work on open-ended problems, make mathematical discoveries, and build mathematics for themselves.

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How might students' conceptions of what mathematics is differ if their experiences with the discipline involved the opportunity to ask questions and follow them on a quest of discovery?

MATHEMATICS AS AN EVOLVING FIELD

Unfortunately, many leave high school and college thinking of the field of mathematics as closed—that is, that mathematics was developed in the past and that there are no current, unsolved problems on which people are actually working. Mathematics is very much a living, creative, and evolving field. While much of what is taught at the undergraduate level, and prior, is centuries old and very well established, there are ways to highlight the evolving nature of mathematics, to talk about open problems, and to craft curriculum in such a way that allows students to see mathematics as filled with discovery, creativity, and imagination. For those who are interested, there are many journals, books, and online resources that provide open-ended, nontraditional creative problems for students and others to play around with and solve. As an example, the Association of Mathematics Teachers of New York State (AMTNYS) publishes the *New York State Mathematics Teachers' Journal* that features a problems section and accepts solutions for publication. It includes a set of problems called (*not so) elementary* where solutions are accepted only from elementary school teachers and precollege students. One of the most rewarding experiences I have had at the college level is working with two students who persisted through a problem posed in this journal. The students spent months working on the problem, trying new things, and coming at it with fresh ideas time and time again. Finally, they saw a way through, and the joy on their faces when they came to me with their plan for a solution—which they were certain would work—was wonderful, as was the excitement they brought with them the day they saw their solution published in the journal.

There is also a whole area of mathematics known as recreational mathematics, which gathers accessible yet exciting problems and topics for those who have an interest in them. Even a museum has been dedicated to bringing the joy of mathematics to the public. The National Museum of Mathematics opened its doors in New York City on December 15, 2012, and features exhibits that highlight the creativity, utility, and beauty of the subject in ways that are accessible to both children and adults. More information about the museum can be found at https://momath.org. MoMath, as it is known, also offers events one can participate in both in person and virtually, challenging the public to engage in mathematics in new and creative ways.

Resources: Recreational Mathematics

Averbach, B., & Chein, O. (2012). *Problem solving through recreational mathematics*. Courier.

Gardner, M. (2001). *The colossal book of mathematics: Classic puzzles, paradoxes, and problems: Number theory, algebra, geometry, probability, topology, game theory, infinity, and other topics of recreational mathematics*. W. W. Norton.

Gaskins, D. (n.d.). Math adventures for all ages. *Let's Play Math.* https://denisegaskins.com/internet-math-resources/math-adventures-for-all-ages/

Mathematical Association of America. (2022). *Recreational mathematics* [List of book reviews]. https://www.maa.org/tags/recreational-mathematics

O'Beirne, T. H. (2017). *Puzzles and paradoxes: Fascinating excursions in recreational mathematics*. Courier Dover.

Stewart, I. (2009). *Professor Stewart's cabinet of mathematical curiosities*. Basic Books.

DOES MATHEMATICS REQUIRE BRILLIANCE?

There is a related construct here, and that is the idea that being good at mathematics means having a natural talent or ability in the area that somehow cannot be developed through effort and a commitment to study alone. This is quite problematic, as it might keep some from pursuing the discipline while upholding an elitist view of mathematics as reserved for few instead of open to all. Yet, in a study of 1,800 academics who were asked to list the qualities needed to succeed in their discipline, mathematicians were among those who valued brilliance the most (Leslie et al., 2015). Brilliance is typically considered to be something that cannot be taught (Bian et al., 2018; Rattan et al., 2012). Further, mathematicians were more likely to attribute the success of women in their discipline to hard work, while attributing that of men to natural ability or brilliance (Leslie et al., 2015). Imagine sitting in a mathematics course where your instructor believes that to be successful in the field you need a natural ability, which that instructor feels cannot be taught.

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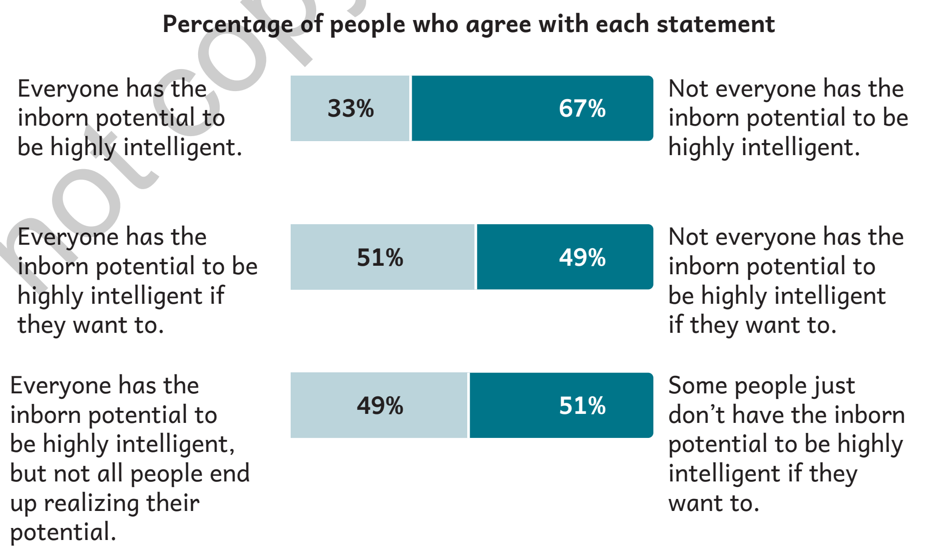
Next, imagine being a woman in this class who shows some success in the material but whose success is attributed by her instructor to effort alone and not to the natural brilliance the instructor feels is needed to be successful in the field. How supported do you think you might feel? How supportive might your instructors be of you if they held such beliefs? Yet this is what occurs, sadly, in far too many classrooms. The characterization that guides the beliefs of the instructors in this study goes beyond those instructors and their classrooms. These are not isolated cases, but rather part of a larger societal construct around how brilliance and mathematics connect. In their work, Meredith Meyer and her colleagues (2015) note that laypeople believe certain fields to require innate talent and that one of these fields is mathematics, so it is not just mathematics professors who hold this belief. This is one of the reasons why so many people feel comfortable admitting they are not good at it. Further, in Western societies, the belief exists that it is not possible for everyone to develop high intelligence. Aneeta Rattan, Krishna Savani, and their colleagues (2012) explored individual perceptions of the potential to develop high intelligence. They gave participants two related statements and asked them to choose which in each pair they agreed with most. As can be seen from Figure 1.3, over two thirds of those asked agreed with the statement that not everyone has the inborn potential to become highly intelligent, as opposed to the statement that everyone does have that potential, no matter how much they may desire to do so (Rattan, Savani, et al., 2012).

Figure 1.3 • *Perceptions of Potential for Intelligence*



This viewpoint cements the idea that either you are good at mathematics or you are not, as there is no way to acquire the high intelligence that many believe is required for success in the field. All of this has implications for how students are, or are not, supported in mathematics learning and for society in general. We need to think about how to counter societal beliefs so that future generations see mathematics as more inclusive and so that we can support more diverse voices among its ranks.

The idea that to be good at math requires a natural brilliance that few have is supported, in part, by the media, which often use mathematics to highlight the intelligence of the characters portrayed or the difficulty of the work they are undertaking. When a student is struggling to do well on an exam and the camera moves from the character to the paper in front of them, it is most certainly a mathematics test they are working on. Mathematical symbols fill the chalkboard behind the emerging scientist who despite all odds, quickly and with certainty, manages to solve the equation that saves the day. Do you ever wonder why the exam is not in history or the chalkboard covered with musical notes?

If we believe brilliance is necessary for success in mathematics, that brilliance is limited to few, and that brilliance cannot be taught, it is difficult to imagine a reality where many decide to study mathematics in our society.

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Further, brilliance, or natural talent, is most often associated with men. Consider that the phrase "Is my son gifted?" is googled two and a half times as often as "Is my daughter gifted?" (Stephens-Davidowitz, 2014). What does this say about the way we view our daughters? What does this say about our expectation that women can and will excel in mathematics? It is, again, hard to imagine that in a society in which these beliefs are prevalent, large numbers of women will be successful in mathematics.

Related to this is the idea that not everyone *can be* good at mathematics. This differs from the belief that not everyone *is* good at mathematics presently, but with effort, training, and the right support one can grow to be good at mathematics. To believe that one can grow to be good at mathematics is to believe that ability in the subject is not fixed. People who have this belief have what is called a growth mindset. A growth mindset allows for someone who might struggle presently with the subject to understand that they can with study and persistence improve their ability. Rattan, Good, and Dweck (2012) conducted a study of mathematics instructors at a competitive private university on the West Coast of the United States. They found that instructors who held a fixed mindset about mathematics were more likely to view those who scored below 65% on their first exam as *bad at math*. This is after just one exam. Keep in mind that these were students at a competitive private university, so they had to have already demonstrated some academic ability to have been admitted. Despite this, instructors with a fixed mindset not only were more likely to view students who scored below a 65% as *bad at math* but also expressed significantly lower expectations for these students when compared to faculty who had a growth mindset. These instructors were also "more likely to comfort students for their (presumed) low ability, and more likely to use teaching strategies that are less conducive to students' continued engagement with the field" (Rattan, Good, & Dweck, 2012, p. 734). By "comforting students," what the authors mean is that these instructors were more likely to tell students that their low ability was okay because not everyone can be good at math. These instructors were also more likely to counsel students out of the discipline and encourage them to study something else. Further, the students who received this feedback were fully aware of the fact that their instructors didn't believe in them and reported that their instructor was more likely to have lower engagement in their learning going forward in the course. Thus, these students weren't getting the support that they needed to do well in the subject, but they *were* getting the clear message that they were not expected to do well based on one exam alone. On the other hand, instructors with a growth mindset were more likely to employ strategy feedback with students who scored below a 65% on their first exam. These instructors were more likely to recommend tutoring, to talk to students about changing their study habits, and to commit to calling on them more in class and giving them more challenging tasks to support their learning. An extensive review of the research on teachers' and students' mindsets and their relationship to student achievement was conducted by Junfeng Zhang and colleagues at the University of Helsinki in 2017. They found that research conducted with school-aged children demonstrates that teacher mindset plays a role in the development of student mindset and as a result impacts student achievement in positive ways. Put simply, teachers with a growth mindset have the potential to positively impact their students at all grade levels.

The belief that mathematics is innate narrows the pool of those who feel comfortable pursuing it. Imagine the discoveries in mathematics that could be made if more people were involved in the development of the discipline. What if more diverse individuals were studying, crafting, and building mathematics? What could be accomplished then? That mathematics is the foundation of so many other disciplines and the basis for the technology on which we rely is well established. The better we are at encouraging more individuals and those from a wider array of backgrounds to study it, the better we will be able to grow and develop our technology and so much of what we depend on in society whether technological or not. Until we reframe what success in mathematics is and how one acquires it, we cannot change a system of mathematics education that privileges few and leaves out too many.

To explore what it means to be good at mathematics in the classroom:

- Have students write their thoughts about how one becomes good at something (including math), then open a conversation about perseverance, practice, and effort.
- Discuss specific skills for improving mathematics ability such as ways of studying and how to read or use a math textbook with your students.
- Share research on learning with your students.
- Remind students and yourself that everyone can learn math to a high degree.
- Give students the opportunity to revise their work, highlighting that what matters is learning over time.

WHAT CAN YOU DO TO CHALLENGE WHAT IT MEANS TO BE GOOD AT MATH?

Part of this work starts with rethinking how we define what it means to be good at mathematics. What a difference one's mindset about what it means to be good at mathematics can make. Let's strive to remind students, colleagues, families, and others we engage with that even if one does not excel in mathematics at present, all have the potential to, given the right supports.

There are several steps we can take to challenge the prevailing beliefs that exist around what it means to be good at mathematics, including engaging students in a discussion around this concept. Others involve

- uncovering our own biases,
- using rich problem-solving tasks,
- normalizing productive struggle, and
- promoting a growth mindset.

Uncovering Our Own Biases

We all have biases. Whether we admit that to ourselves or not, there are beliefs we hold that impact how we experience the world, how we interact with others, and how we engage in the teaching and learning of mathematics. We do well to try and unpack these biases so that we are more aware of them and able to consider how they impact our work. This is the first step in uncovering and nurturing our own strengths as educators (Berry, 2008). One way to start to uncover and interrupt our own biases around students' learning of mathematics is to look at students through a strengths-based lens (Kobett & Karp, 2020). Consider the statements in Figure 1.4, which may reflect things you have commonly heard or maybe even said. What is the underlying belief each statement might convey? How can reframing that statement through a strengths-based lens help turn the bias around?

As Beth Kobett and Karen Karp (2020) share, "The first step in changing the narrative is to consciously hear the language that we and others use to describe our students. When we work to identify a belief that focuses on moving in a positive direction, we are more likely to interact with other teachers, our students, and families in more positive and productive ways" (p. 28).

Now it's your turn to consider your beliefs about your students: Try visualizing a student who you think is good at math, and write down concrete reasons why you believe this to be true. What about a student who you think is struggling? What words have you used to describe this student in the past, and what might those statements convey about your underlying beliefs? What does this tell you about how you conceptualize what student success looks like in mathematics? How might you expand or challenge your reasons?

Figure 1.4 • *Shifting to Beliefs That Emphasize Students' Strengths*

Statement	Underlying Belief	Alternative Belief
She is doing the best she can.	She can't learn more.	She can learn math. We just need to find an entry point into her learning.
He can't help it.	He doesn't have self-control or self-regulation skills.	He has better self-control when he is able to select manipulatives, tools, and a place to work.
We can't expect her to do more.	She cannot learn more mathematics than she is currently learning. She is incapable of learning more.	If we raise our expectations and set success criteria in collaboration with the student, she will be able to achieve.
She lacks the background knowledge to grasp this information.	How much students can learn depends on the background knowledge they hold. Students are unable to learn without the right background knowledge.	She has solid knowledge about money. Let's use that knowledge to develop ideas about place value.
She does not care.	This student's behavior indicates that she does not value school.	The student's behavior indicates that we need to show her how much we care about her learning.
Even though he is motivated to learn, he is unable to retain the concepts.	Students who struggle with retention cannot learn mathematics.	I notice that he retains more when he is able to work with his peers to solve problems. Let's try pairing him with a classmate.
He can do this—he is just lazy.	Students choose to not work.	We need to find out why he does not complete his work.
You just need to tell them how to do it because they can't think on their own.	Direct instruction is best for students who struggle.	My students are capable of higher-level thinking and problem solving.
His parents don't care and can't help him.	Families that cannot attend conferences don't care about their child's learning.	Families care very much about their children's school success, but don't always show it in the same way or in ways that resonate with teachers' own families.

SOURCE: Kobett, B. M., & Karp, K. S. (2020). *Strengths-based teaching and learning in mathematics: Five teaching turnarounds for Grades K–6*. Corwin. Reprinted with permission.

Resources

- Check out the self-assessments in Chapter 1 of Kobett and Karp's text at resources.corwin.com/badatmath to learn about your mathematics identity as well as the beliefs you hold about yourself as a mathematics teacher and your students as learners of mathematics.
  - Kobett, B. M., & Karp, K. S. (2020). *Strengths-based teaching and learning in mathematics: Five teaching turnarounds for Grades K–6*. Corwin.
- Take Harvard's Project Implicit online assessment to identify attitudes, beliefs, and implicit biases around race, gender, and other areas.
  - Project Implicit. (2011). *Preliminary information: Implicit Association Test (IAT).* https://implicit.harvard.edu/implicit/takeatest.html
- Write out what you think it means to be good at mathematics. Review your words and consider how your definition affects your work as a teacher, instructional leader, or administrator.

Using Rich Problem-Solving Tasks

One way to promote the view that being good at mathematics isn't about having an ability to compute quickly is to have students work on problems that are rich, complex, and open-ended, where not all the required symbolism, language, and steps are predetermined and have been previously taught. That is, you can have students build mathematics appropriate to their grade level by working on problems and activities that require them to do some research, employ some tools, and really see the creativity involved in the work. This does not have to be done exclusively, and certainly there is still a place for more traditional textbook problems, but too often these rich, open-ended problems are excluded from students' school experiences, especially for students in special education classes. Some believe that until students can master the textbook problems, they cannot engage in the higher-level thinking required for these more open-ended, rich problems, but the opposite is true. Until students have the opportunity to build mathematics and engage in rich open-ended problem solving, they may not solidify their understanding of the more traditional problems and may grow frustrated by a series of problems disconnected from themselves with preset steps they need to memorize rather than fully understand. They don't get at the *why*—why the method works—and they don't own the mathematics. They fail to see how a seemingly simple question can lead to an amazingly rich array of mathematics that they can have a hand in building with teacher guidance, but also with their own voices.

Further, when given a set of prescribed steps or guidance that overscaffolds a problem, students become less able to successfully work through problems that differ from those they are typically given. This is a case where less is more. Less scaffolding and less structure might afford students the opportunity to play with the mathematics in ways that are creative and allow them to think through the problem differently. In a study of two schools, one that used open-ended tasks in classes and one that used more traditional approaches, Boaler (2002, 2015a) found that in the school that used open-ended problems students who worked together toward a solution were better able to problem solve and to work through problems that differed from those they had previously seen. They were also more likely to say they enjoyed mathematics and years later were more likely to be in highly skilled and professional careers as compared to the students at the more traditional school.

Even the youngest of children can investigate patterns and draw inferences from these patterns. Students learning multiplication can look at multiples of 5 or 10 and draw conclusions about them, or they can investigate whether multiplying by an even number always yields an even number or whether multiplying by an odd always yields an odd and maybe start to get at why. Open-ended questions about the relative speed at which two vases of different shapes will fill when water is poured into them at the same rate can spark talks about volume, shape, rates, and measurements with little but a question posed. Students can be given the supplies needed to investigate these situations and begin to make predictions not just about which vase will fill first but about what properties matter and whether there is a way to tell in certain cases without much work. Exploring in which vase the water level rises fastest is a related problem that is worth considering as well. These questions and explorations are quite far removed from traditional textbook problems but can, in fact, teach the same concepts. One thing you can try in preparation for your next class is to rewrite a closed question you might have planned to use with your class as an open question.

Resources: Open-Ended Rich Problems

Dougherty, B., & Venenciano, L. (2023). *Classroom-ready rich algebra tasks, Grades 6–12: Engaging students in doing math.* Corwin.

Fletcher, G. (n.d.). *3 Act Task file cabinet.* https://gfletchy.com/3-act-lessons/

House, P., Stenglein, S., & Day, R. (2021). *Activity gems for the 9–12 classroom.* National Council of Teachers of Mathematics.

Kobett, B. M., Fennell, F., Karp, K. S., Andrews, D. R., Knighten, L. D., & Shih, J. C. (2021). *Classroom-ready rich math tasks, Grades K–1: Engaging students in doing math*. Corwin.

Kobett, B. M., Fennell, F., Karp, K. S., Andrews, D. R., & Mulroe, S. T. (2021). *Classroom-ready rich math tasks, Grades 4–5: Engaging students in doing math*. Corwin.

Kobett, B. M., Karp, K. S., Harrison, D., & Swartz, B. A. (2021). *Classroom-ready rich math tasks, Grades 2–3: Engaging students in doing math*. Corwin.

Open Middle Partnership. (2022). *Open Middle: Challenging math problems worth solving.* www.openmiddle.com

Orr, J., & Pearce, K. (Hosts). (2018–present). *Making math moments matter* [Audio podcast]. https://makemathmoments.com/podcast/

Smith, M., Bill, V., & Steele, M. D. (2020). *On your feet guide to modifying mathematical tasks.* Corwin.

Stanford Graduate School of Education. (n.d.). *YouCubed®: Inspire ALL students with open, creative, mindset mathematics.* Stanford University. www.youcubed.org

Normalizing Productive Struggle and Supporting a Growth Mindset

Let us normalize struggle in the classroom and beyond. It is often the case that when a student is struggling on a problem, our instinct is to help them by nudging them along through the work. I know I too am guilty of doing this, but it feeds into the idea that if one does not get a problem right away, something is wrong, and one needs help immediately. What if instead, as a classroom teacher, you gave students the time and space needed for them to struggle? What if as an instructional leader you promoted the idea that struggle is positive? What if it became the norm that we might have to try a problem more than once and in multiple ways to understand it?

If we normalized taking multiple approaches and at times multiple attempts to solve a problem, we would be building perseverance in our students, allowing them to engage in productive struggle and teaching them that what matters is not getting something quickly. Those students who are able to stick with the discomfort of not knowing and play with the math for longer despite it tend to do better, so why not cultivate this in students? I am suggesting not that we never step in to help but rather that we delay that help a bit in cases where we know that students—with more time, more approaches, and a willingness to stick with a problem—can in the end reach a solution if they believe that this will come with time, albeit not always with comfort. How much more resilient might students be if they realized that a stumble is just that and not a judgment on their abilities or a prediction of future failure?

One thing you can try in your next class is to ask a student who has solved a problem to solve it a different way and see what arises. The fact that mathematics problems can be tackled in so many ways yet reveal the same reality is amazing to me and something that we should value more in our teaching. When a student completes a problem and arrives at the right solution, we often end there. What if instead we asked that same student to consider a different path to the solution? How much more creative and versatile might their understandings be if students were pushed to look for multiple paths? In fact, mathematicians often do this. Perhaps they prove a particular theorem in some way. Then, they consider finding a more elegant proof, one that uses fewer steps, perhaps is more direct or more efficient in its approach, or has some other feature that is of value in some way even if that way is a matter of aesthetics. Develop in the budding mathematicians in your classes the ability to look at the same set of information in new ways: perhaps an algebraic approach followed by a geometric approach, or a numeric approach followed by a visual one. The Pythagorean theorem, as an example, has more distinct proofs than any theorem in history. In her 1968 text, *The Pythagorean Proposition*, Elisha Scott Loomis includes no less than 367 such proofs, including one by former U.S. president James Garfield. Being able to find another way is a valuable skill in life, so it seems useful to include this inclination as part of the way in which we teach mathematics.

Additionally, let us occasionally give students problems that we know will take a long time to solve, that will require multiple attempts, and that they perhaps will not be able to solve in the end. This will help normalize the idea that true mathematical work takes time and struggle and does not always yield a solution as we might have wanted. This is fine. It is the reality of the field and one students will benefit from knowing about. If we want people to fundamentally change the way they conceptualize who is good at mathematics, we must give students fundamentally different classroom experiences.

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One way to do this is to ensure that students have ample opportunities to work together on tasks and activities. Mathematicians often collaborate with others. In fact, conferences are set up with working groups where mathematicians of certain specialties gather to work on open problems in particular fields together. Developing mathematics is, in many cases, a social process. Let us ensure that our students have the opportunity to see and experience it as such.

Resources: Growth Mindset and Productive Struggle

Brock, A., & Hundley, H. (2018). *In other words: Phrases for growth mindset: A teacher's guide to empowering students through effective praise and feedback*. Simon & Schuster.

SanGiovanni, J. J., Katt, S., & Dykema, K. J. (2020). *Productive math struggle: A 6-point action plan for fostering perseverance*. Corwin.

In this chapter we considered people's perceptions around brilliance and its connection to mathematics. Here, we have much work to do. It is troubling to know that half of those asked believe that there are some individuals who lack the ability to do well in math and who despite their efforts cannot do well in it. This is flatly unacceptable. It erodes efforts at making the discipline one in which all can succeed, and it exemplifies a fixed mindset that is harmful to the people who hold these beliefs. We need extensive professional development around the idea of growth mindsets. Rather than counseling them out, we need to actively work with students who with some support would excel, and we need to ensure that more women and members of underrepresented groups in mathematics are supported rather than excluded not only by developing programs to support them but by actively changing the culture so that in time these programs are not needed. We must talk to students from an early age about growth mindsets and instill in them the belief that their ability to learn and grow is not fixed. You can do this both in and out of the classroom. When those around us talk in ways that support fixed mindsets or the idea that not all of us can excel in mathematics, we must challenge them with vigor. Remind them that "no one is born lacking the ability to learn math" (Boaler, 2015a, p. 5). Actively talk to your students about the ways in which they can grow as learners of mathematics. Consider also the feedback and praise you give to students and how your words reinforce or challenge the idea that intelligence is fixed. Praise is often used in classrooms as an effective tool, but it can reinforce fixed mindsets if we, for example, praise students for being *smart* or *having smart ideas*. Instead, try praising students for things they can control such as working hard on a task or trying multiple approaches. "Praise feels good, but when people are praised for who they are as a person ('You are so smart'), rather than what they did ('This is an amazing piece of work') they get the idea they have a fixed amount of ability" (Boaler, 2015a, p. 8). In the feedback you provide, you can model and support the idea that intelligence is something one can grow and develop over time.

Questions for Reflection

For Teachers

- What messages do you send to students about who is good at mathematics through your words, actions, the way you decorate your classroom, and the resources you employ?
- How might you bring open problems and the work of mathematicians into your teaching?
- What rich tasks can you employ to teach mathematical content?
- How can you talk to parents and others who may believe that it is socially acceptable to be bad at math?
- What stories did you believe about mathematics when you were a student? What stories do you believe now?

For Instructional Leaders

- In what ways do the activities you use with teachers promote the idea that to be good at math is to persist through problems?
- How do the resources you provide challenge traditional views of what it means to be good at math?
- How do you respond when someone admits to you that they are bad at math?
- How do you put into practice the idea of growth mindsets in your own work with teachers?
- What beliefs do you hold about what it means to be good at mathematics? How do these influence your work with teachers?

For Administrators

- How do the curriculum materials and resources your school adopts speak to what it means to be good at mathematics? How do they allow for productive struggle? How do they engage students in open-ended problem solving?

- What opportunities do you create for teachers, parents, and community members to grapple with the ideas in this chapter? Are there intentional spaces created for these individuals to come together to discuss ideas?
- How can you encourage productive struggle among your teachers by allowing them the space and time needed to try different teaching techniques and approaches to find what works best for them and their students?
- What beliefs do you hold about what it means to be good at mathematics? How do these affect your decisions around curriculum, hiring, and teacher development?