PART I

Introduction to Teaching and Learning Mathematics

Chapter 1

Introduction to the Field of Mathematics Education

The past few decades have seen incredible growth in the study of teaching and learning mathematics. K–12 teachers, university professors, and other educators have produced standards documents, research reports, and curriculum frameworks with the potential to help improve students' learning. All of this activity makes it an exciting time to enter the profession of mathematics teaching. However, it can also be overwhelming to try to digest and reflect on everything the field has to offer. In fact, one is never really done learning about teaching mathematics. The best teachers are always learning ways to improve their practices by talking with colleagues, reading research, reading teachers' journals, carefully assessing the impact of their instructional practices on their students' thinking, and adjusting their practices to maximize students' learning.

The goal of this chapter is to provide a sense of the major issues and trends that have shaped the field of mathematics education in the recent past. By way of introduction, we will examine the standards documents published by the **National Council of Teachers of Mathematics** (NCTM), an organization with more than 90,000 members dedicated to improving mathematics education. We will then examine trends in mathematics teaching and learning around the world and the central messages of the reform movement in mathematics education. The objective is not to completely "cover" or give a comprehensive treatment of each of these topics—volumes have already been written on each of them—and resources for further study are given at the end of the chapter. Instead, the chapter provides a frame of reference for understanding the rest of the text. Remember, the best teachers are those who are always learning, and reading this chapter represents just the first step in a career-long journey of navigating the field.

A Brief History of NCTM Standards

The 1980s and 1990s marked the beginning of the "standards movement" because of the effort put into developing standards for teaching and learning in various subject areas. NCTM released three standards documents during this period: (1) ***Curriculum***

and Evaluation Standards for School Mathematics (1989), (2) ***Professional Standards for Teaching Mathematics*** (1991), and (3) ***Assessment Standards for School Mathematics*** (1995). The major themes from this first round of standards laid the groundwork for a fourth influential document, ***Principles and Standards for School Mathematics*** (NCTM, 2000). To understand the current state of the field of mathematics education, it is important to grasp the central messages conveyed by each document.

Curriculum and Evaluation Standards

NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) described a vision for the teaching and learning of mathematics that differed sharply with much of conventional practice. For example, in regard to algebra, it called for more attention to (1) "developing an understanding of variables, expressions, and equations" (p. 70) and (2) "the use of real-world problems to motivate and apply theory" (p. 126). Less attention was to be given to (1) "manipulating symbols" (p. 70) and (2) "word problems by type, such as coin, digit, and work" (p. 127). The document contained similar direction for other mathematics content areas, including number and operations, geometry, and measurement. The recommendations sought to move school mathematics beyond an exclusive focus on the teaching and learning of procedures. A central emphasis was helping students to understand "the importance of the connections among mathematical topics and those between mathematics and other disciplines" (p. 146).

Curriculum and Evaluation Standards was also revolutionary in its call for more attention to historically neglected areas such as statistics, probability, and discrete mathematics. Although many important applications of these content areas could be found in contemporary society, they were largely absent from the school mathematics curriculum. The recommendation to give more attention to neglected areas was based on the premise that the school curriculum should change as the needs of society change. This premise also dictated that the school curriculum should take advantage of technology to help students understand the conceptual underpinnings of mathematics. In sum, *Curriculum and Evaluation Standards* recommended reform in *what* was taught as well as *how* it was taught.

Professional Standards for Teaching Mathematics

Professional Standards for Teaching Mathematics helped further clarify NCTM's vision for school mathematics reform. It recommended five major shifts in mathematics classroom environments:

- Toward classrooms as mathematical communities—away from classrooms as simply a collection of individuals;
- Toward logic and mathematical evidence as verification—away from the teacher as the sole authority for right answers;
- Toward mathematical reasoning—away from merely memorizing procedures;
- Toward conjecturing, inventing, and problem-solving—away from an emphasis on mechanistic answer-finding;

- Toward connecting mathematics, its ideas, and its applications—away from treating mathematics as a body of isolated concepts and procedures. (NCTM, 1991, p. 3)

NCTM (1991) recognized that these shifts would not occur overnight. Sustained professional development would be necessary to help teachers implement the recommendations.

NCTM's *Professional Standards for Teaching Mathematics* calls for five major shifts in the environment of mathematics classrooms (see the preceding discussion). What changes, if any, would your past mathematics teachers in Grades K–12 have needed to make to align their instruction with the five recommendations? Provide specific examples.

STOP TO REFLECT

Assessment Standards for School Mathematics

Assessment Standards for School Mathematics marked the end of the first round of NCTM standards documents. The document defined assessment broadly as "the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes" (NCTM, 1995, p. 3). From this perspective, one of the primary purposes of assessment is to provide teachers information about the nature of student learning. Information about students' learning can be drawn from a variety of sources. Instead of relying solely on paper-and-pencil tests, teachers can draw information from student interviews, projects, and portfolios. Information gained about students' learning can in turn help shape future lesson plans.

Principles and Standards for School Mathematics

Principles and Standards for School Mathematics (NCTM, 2000) differed from previous standards documents in that its intent was to write standards that

- build on the foundation of the original *Standards* documents;
- integrate the classroom-related portions of *Curriculum and Evaluation Standards for School Mathematics, Professional Standards for Teaching Mathematics*, and *Assessment Standards for School Mathematics;*
- organize recommendations into four grade bands: prekindergarten through Grade 2, Grades 3–5, Grades 6–8, and Grades 9–12. (p. x)

Principles and Standards for School Mathematics organized its discussion of mathematics content around five content standards: number and operations, algebra, geometry, measurement, and data analysis and probability. The second half of this text uses a similar organizational scheme by devoting chapters to each of the content standards (with the exception that measurement is distributed among the other content strands).

As a consolidation and elaboration of the previous NCTM standards documents, *Principles and Standards for School Mathematics* represents the closest we have come to a consensus about *which* mathematical topics should be taught in school and *how* they should be taught. Teachers, university professors, mathematics supervisors, and other professionals spent three years constructing the document. As it was being written, feedback was elicited from stakeholders in mathematics education around the world. It should be noted, however, that consensus on the vision of NCTM standards has never been, and likely never will be, universal. For example, some disagree with the decreased emphasis on lecture as a teaching method (Wu, 1999b) or the manner in which technology is to be integrated into the curriculum (Askey, 1999). These kinds of conflicts have been characterized as parts of a larger "math war" over the content of the school curriculum (Schoenfeld, 2004). As with most events characterized as "wars," much of the conflict is based on the opposing sides misunderstanding each other. This book is based on the premise that one must seek to understand the NCTM standards before condemning *or* accepting them. Toward that end, in the next section, an overview of some of the most important themes in *Principles and Standards for School Mathematics* is given.

Understanding the Principles and Process Standards

Principles and Standards for School Mathematics goes beyond merely providing an organizational scheme for discussing mathematics content. The document also contains principles and process standards to guide the teaching of mathematics. **NCTM principles** "describe particular features of high-quality mathematics education" (NCTM, 2000, p. 11), while **NCTM process standards** describe aspects of mathematical teaching and learning that should occur in all content areas. The six principles are Equity, Curriculum, Teaching, Learning, Assessment, and Technology. The five process standards are Problem Solving, Reasoning and Proof, Communication, Connections, and Representation.

NCTM Principles

The Equity Principle states that "excellence in mathematics education requires equity—high expectations and strong support for all students" (NCTM, 2000, p. 12). This challenges the assumption that only an elite few are meant to understand mathematics. Teachers should expect *all* students to learn mathematics. The principle calls for equity of access to high-quality mathematics instruction, curriculum materials, and technology. A key to understanding the Equity Principle is the premise that "equity does not mean that every student should receive identical instruction[;] instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (NCTM, 2000, p. 12). Therefore, as teachers think about how to achieve equity in their own classrooms, they need to set aside the assumption that "all students should be treated the same way."

The Curriculum Principle states that "a curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well-articulated across the grades" (NCTM, 2000, p. 14). To understand this principle, it can be helpful to consider a counterexample: some curricula are essentially "laundry lists" of isolated "topics" to be "covered" in a prescribed order. The order in which topics are treated may or may not help students understand the fundamental concepts of the subject. Teachers in such situations may rarely, if ever, plan sequences of instruction with colleagues who teach different grade levels or subjects. The end result is that students perceive mathematics to be a disconnected body of knowledge consisting of isolated rules and procedures that do not relate to one another in a coherent fashion. To avoid this situation, it is important for teachers and instructional supervisors to communicate with one another about student learning and to modify curriculum sequences as necessary.

The Teaching Principle states that "effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000, p. 16). To pursue this principle, one must develop knowledge of students, knowledge of mathematics, and knowledge of teaching strategies (Kilpatrick, Swafford, & Findell, 2001). Developing knowledge of students' mathematical thinking allows teachers to understand the effectiveness of their instructional practices. Developing knowledge of mathematics helps teachers identify the "big ideas" in any given domain and to draw students' attention toward them. Knowledge of teaching strategies gives teachers a variety of instructional practices that can be adapted, as needed, to any given situation. A teacher is never done developing knowledge in any of the three areas. It is important to constantly seek out opportunities to develop and refine knowledge in each area.

The Learning Principle states that "students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000, p. 20). It is not enough for students to memorize mathematical procedures and skills in isolation from one another. While procedures and skills are important, it is unlikely that students will be able to apply them flexibly if they do not understand the big picture of why they work and how they are related. Many students have experienced "learning" a set of facts or skills needed for a test and then forgetting them shortly after taking it. Often this occurs because the facts and skills were not connected to a larger mental map of the structure of the discipline. The goal should be to help students build rich, interconnected mental roadmaps of a given content area so that they will progress beyond surface-level learning that is easily forgotten after a test has been taken.

The Assessment Principle asserts that "assessment should support the learning of important mathematics and furnish useful information for both teachers and students" (NCTM, 2000, p. 22). This principle amplifies the view of assessment given in the *Assessment Standards for School Mathematics* by emphasizing the use of assessment results to inform teaching. It also reiterates the importance of thinking beyond traditional paper-and-pencil tests for assessment. Teachers should gather evidence of students' learning from multiple sources, including "open-ended questions, constructed response tasks, selected-response items, performance tasks, observations, conversations, journals, and portfolios" (NCTM, 2000, p. 23). Insights gained from listening to students as they work with one another or reading what they write in response to a prompt from the teacher can be invaluable in improving the effectiveness of instruction.

The last of the six principles is the Technology Principle, which states that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 24). Properly used, technology can help students develop the type of conceptual understanding described in the Learning Principle. Instead of eroding computational skills, it can help students understand the central ideas of a given content area. Of course, if one's instructional goal is merely to teach students how to perform computations, then current technology can, in fact, seem threatening. However, since instruction needs to go beyond that sort of surface-level treatment, it is important to think about how technology can help students develop deeper understandings of important mathematical ideas. The Technology Connections included throughout this book provide specific examples of how technology can facilitate the teaching and learning of mathematics in meaningful ways.

STOP TO REFLECT

From your own experiences as a K–12 student, write an example of how schools sometimes attain each of the NCTM principles described above. Then, once again drawing on your own experiences as a K–12 student, write an example of how schools sometimes fall short of the ideals set forth in each principle. Share and compare examples with classmates.

NCTM Process Standards

Problem Solving is the first of the process standards discussed in *Principles and Standards for School Mathematics.* Several aspects of this process standard conflict with traditional notions of problem solving in mathematics. The process standard defines problem solving as "engaging in a task for which the solution method is not known in advance" (NCTM, 2000, p. 52). Under this definition, students who simply apply procedures they have been taught to produce solutions to sets of homework problems are not engaging in problem solving. In addition, problem solving should not be thought of as just doing the "application" problems that often appear at the end of a set of exercises. Students are to build mathematical knowledge *during* the process of problem solving. As they do so, they develop the attitude that mathematics makes sense and is a matter of reasoning carefully through novel situations rather than simply following teacher-prescribed recipes. Teaching via problem solving is arguably *the* central idea in reform-oriented instruction, and sometimes one of the most difficult to grasp. We will return to it at the end of this chapter and throughout the book.

Reasoning and Proof is the second process standard. While this standard sets the goal that students should be able to produce formal proofs by the end of high school, it deals with much more than formal proof. Teachers can help students develop mathematical reasoning ability simply by asking *why* a given procedure works or *how* they produced the solution to a question. Pragmatically speaking, teachers need to help students get in the habit of answering these types of questions on account of the numerous standardized tests that now include open-ended questions asking for justifications and explanations of answers. However, students' performance on tests is not the most compelling reason to help them develop the ability to reason and prove. If students do not engage in some form of reasoning, it is arguable that they are not doing mathematics at all. The goal should be to develop a classroom community in which students are curious to find out why things work as they do and to make and test their own conjectures about solutions to problems.

The third process standard is Communication. As students communicate their reasoning with peers and teachers by writing and talking, they are forced to structure their thoughts. In this process, they build and refine their mathematical knowledge. Classroom communication patterns need to go beyond teacher-student interaction alone. Student–student interaction is also important, because it gives students the opportunity to "analyze and evaluate the mathematical thinking and strategies of others" (NCTM, 2000, p. 60). The teacher's responsibility is to create a classroom environment in which students feel free to communicate their thinking and respect and value one another's contributions. This is a nontrivial task, particularly if students are not used to sharing their mathematical thinking with one another. It is, nonetheless, a valuable goal because of the opportunities it affords to help students reflect on and subsequently refine their thinking.

The fourth process standard is Connections. As stated in the Learning Principle, students should see connections among mathematical ideas rather than viewing mathematics as a subject consisting of isolated topics. For example, teachers should ask questions and present problems that help students see how fractions, decimals, and percents are related to one another. A counterexample to this would be dealing with fractions, decimals, and percents in separate chapters and never drawing students' attention to the fact that they are often used as different representations of the same quantities. A second kind of connection students should see is how the mathematics they are studying applies to contexts outside of mathematics. For example, students can be asked to solve problems that connect data analysis, probability, and mathematical modeling to making predictions about the weather or other physical phenomena. By paying careful attention to the problems they pose, teachers can systematically begin to help students see the connections among mathematical ideas as well as how those ideas connect to other fields.

The final process standard is Representation. Representation may not come to mind immediately as an important mathematical process simply because it is easy to take representations for granted. For example, when we use our place value system for representing numbers, we seldom stop to reflect on how much more efficient it is than using Roman numerals. It is important to realize, however, that any given representation is likely to be interpreted differently by individual students. Those without a deep understanding of place value may not see the efficiency in our number system. Teachers need to work to understand how their students interpret representations and design instruction to be responsive to students' needs. To facilitate learning, it is necessary to help students build bridges from their intuitive, idiosyncratic representations for concepts to conventional representations. Technology affords unique opportunities for students to explore conventional representations. For example, graphing calculators allow one to efficiently move back and forth between graphical and tabular representations of functions. Instead of simply asking students to produce graphs and tables using predetermined symbolic representations of functions, teachers can ask students to explain how specific aspects of each representation relate to one another. As students consider multiple ways to represent mathematical ideas, they can reflect on which representations are most efficient for different situations.

STOP TO REFLECT

From your own experiences as a K–12 student, write an example of how schools sometimes attain each of the NCTM process standards described above. Then, once again drawing on your own experiences as a K–12 student, write an example of how schools sometimes fall short of the ideals set forth in each standard. Share and compare examples with classmates.

Common Core State Standards

Many of the recommendations from NCTM's standards documents are reflected in the ***Common Core State Standards*** (CCSS; National Governor's Association for Best Practices & Council of Chief State School Officers, 2010). These standards were written to provide common mathematics learning expectations across the United States. Forty-eight states participated in writing the CCSS, and the vast majority of these states also adopted them (see www.corestandards.org for information about your state). The CCSS are similar to NCTM's *Principles and Standards for School Mathematics* in that they provide standards for the content as well as the processes of learning mathematics. CCSS process and content standards relevant to middle and high school mathematics appear in Appendices A, B, and C of this text and are also available online at www.corestandards.org. Throughout this book, you will find notes in the margins pointing toward tasks to help develop your ability to implement the CCSS standards for mathematical practice and the CCSS content standards for Grades 6 through 12.

CCSS Standards for Mathematical Practice

According to the CCSS standards for mathematical practice, students should experience the following mathematical thinking processes in the classroom:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning. (National Governor's Association for Best Practices & Council of Chief State School Officers, 2010, p. 53)

A description of each practice is provided in Appendix A.

STOP TO REFLECT

Read the description of each of the CCSS standards for mathematical practice in Appendix A. Describe how each practice connects to one or more of the NCTM process standards.

This book includes classroom vignettes to spark thought and discussion about how each of the CCSS standards for mathematical practice can be enacted in the classroom. The vignettes do not present images of classrooms perfectly aligned with the process standards and standards for mathematical practice. Instead, they present realistic situations vividly illustrating the challenges teachers encounter on a daily basis in trying to attain the standards. Reading the vignettes and discussing them with others is a good starting point for thinking about how to overcome the challenges.

Implementing the Common Core

See the clinical task at the end of the chapter to analyze the extent to which a lesson aligns with all of the CCSS standards for mathematical practice.

CCSS Content Standards

The CCSS contain five content domains for middle school and six for high school. For Grades 6 through 8, the five content domains are ratios and proportional reasoning, the number system, expressions and equations, geometry, and statistics and probability. For high school level, the six content domains are number and quantity, algebra, functions, modeling, geometry, and statistics and probability. These content domains closely resemble the content strands of NCTM's (2000) *Principles and Standards for School Mathematics.* The alignment between CCSS content domains and the chapters of this book is shown in Table 1.1.

Table 1.1 Relationship Between *Common Core State Standards* Content Domains and the Chapters in This Book

Content Domains From *Common Core State Standards*	Related Chapters in *Teaching Mathematics in Grades 6–12*
Grades 6–8	
Ratios and proportional reasoning	Chapter 7: Developing Students' Thinking in Number and Operations
The number system	Chapter 7: Developing Students' Thinking in Number and Operations
Expressions and equations	Chapter 8: Developing Students' Algebraic Thinking
Geometry	Chapter 10: Developing Students' Geometric Thinking
Statistics and probability	Chapter 9: Developing Students' Statistical and Probabilistic Thinking
High School	
Number and quantity	Chapter 7: Developing Students' Thinking in Number and Operations
Algebra	Chapter 8: Developing Students' Algebraic Thinking
Functions	Chapter 8: Developing Students' Algebraic Thinking; Chapter 11: Developing Students' Thinking in Advanced Placement Courses
Modeling	Throughout Chapters 7–11
Geometry	Chapter 10: Developing Students' Geometric Thinking
Statistics and probability	Chapter 9: Developing Students' Statistical and Probabilistic Thinking; Chapter 11: Developing Students' Thinking in Advanced Placement Courses

A Global Perspective on Mathematics Education

Most of the introductory material in this chapter has dealt with developments in mathematics education in North America. To understand more fully how mathematics education can be improved in the classroom, it is helpful to examine how mathematics is taught across the globe. Each country has a distinct culture of teaching that shapes students' experiences with mathematics. Typical lessons often vary in important ways from one country to the next. Although the mathematics students are to learn may be the same in any two given countries, the methods of teaching the content can differ sharply.

The **Third International Mathematics and Science Study (TIMSS)** was a large, comprehensive investigation of mathematics teaching and learning in different parts of the world. Conducted in 1994–1995, it tested mathematics achievement in more than 40 nations. Top-scoring countries on the assessment included Singapore, Korea, and Japan, with eighth-grade mean scores of 643, 607, and 605, respectively. Canada, England, and the United States all scored significantly lower, with eighth-grade mean scores of 527, 506, and 500, respectively (A. E. Beaton & Robitaille, 1999). Repeat administrations of the TIMSS in 1999, 2003, and 2007 showed a persistent achievement gap between U.S. students and their counterparts in other industrialized nations (National Center for Education Statistics, 2010). The TIMSS acronym now stands for "Trends in International Mathematics and Science Study."

TIMSS studies have gone beyond just measuring students' achievement. They have also uncovered factors that could explain achievement differences among countries. In a book titled *The Teaching Gap*, J. W. Stigler and Hiebert (1999) analyzed video footage of eighth-grade mathematics lessons from countries participating in the 1994–1995 TIMSS. Their analysis is especially helpful in shedding light on how mathematics is taught differently between the United States and Japan. They summarized the Japanese style of teaching in the following terms:

> In Japan, teachers appear to take a less active role, allowing their students to invent their own procedures for solving problems . . . Teachers, however, carefully design and orchestrate lessons so that students are likely to use procedures that have been developed recently in class. An appropriate motto for Japanese teaching would be "structured problem solving." (J. W. Stigler & Hiebert, 1999, p. 27)

In contrast, the typical pattern of teaching in the United States was described as follows:

> In the United States, content is not totally absent . . . but the level is less advanced and requires much less mathematical reasoning . . . Teachers present definitions of terms and demonstrate procedures for solving specific problems. Students are then asked to memorize the definitions and practice the procedures. In the United States, the motto is "learning terms and practicing procedures." (J. W. Stigler & Hiebert, 1999, p. 27)

Figures 1.1 and 1.2 show sample lessons drawn from the 1999 TIMSS video study. Figure 1.1 represents an American lesson, and Figure 1.2 represents a Japanese lesson. The American lesson illustrates "learning rules and practicing procedures," while the Japanese lesson illustrates "structured problem solving."

Figure 1.1 A typical American mathematics lesson from the TIMSS video study.

[45 minute lesson]

United States Public Release Lesson 4 Lesson Graph [8th grade]

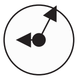
4 1/2 minutes

Private Class Work: Reviewing the Previous Work

Students get their materials for the lesson ready, and then work on the following "problem of the day":

Tell about these in your own words

1) Inscribed angle theorem 2) right angle corollary 3) arc intercept corollary



26 minutes

Public Class Work

Problem of the day results

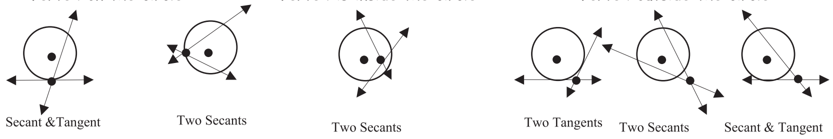
Inscribed angle theorem –Suzy: "If an angle's inscribed in a circle and it intercepts part of the circle, then the angle's measure is equal to half of the other angle."

Right angle corollary –Matt: "If an inscribed angle intercepts a semicircle, the angle is a right angle."

Arc intercept corollary – Margaret: "When you have two inscribed angles and they intercept the same arc, they have the same measure."

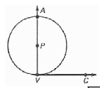
Review of Angles formed by Secants and Tangents

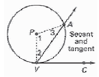
Vertex on the circle **Vertex is inside the circle** **Vertex outside the circle**



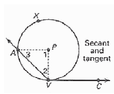
Homework Problems

secant and tangent intersecting on a circle





∠AVC is an acute angle



∠AVC is an obtuse angle

m∠AVC=90

$m\widehat{AV}$=180

$m\widehat{AV}$	m∠1	m∠2	m∠PVC	m∠AVC
120°	120°	30°	*90°*	60°
x°	*x°*	*(180°-x°)÷2*	*90°*	*x°÷2*

$m\widehat{AXV}$	m∠1	m∠2	m∠PVC	m∠AVC
200°	160°	10°	*90°*	100°
x°	*160°-x°*	*(180°-m<1)÷2*	*90°*	*x°÷2*

Theorem: If a tangent and a secant (or a chord) intersect on a circle at the point of tangency, then the measure of the angle formed is half the measure of its intercepted arc – regardless of whether the angle is right, acute, or obtuse.

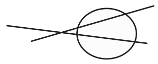
Two secants intersecting inside a circle

$m\widehat{AC}$	$m\widehat{BD}$	m<1	m<2	m<AVC	m<DVB
160°	40°	80°	20°	100°	100°
180°	70°	*90°*	*35°*	*125°*	*125°*
X_1°	X_2°	*mAC÷2*	*mBD÷2*	*(mAC+ mBD)÷2*	*(mAC+ mBD)÷2*

Theorem: The measure of an angle formed by two secants or chords that intersect in the interior of a circle is half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.

Two secants intersecting outside a circle

$m\widehat{BD}$	$m\widehat{AC}$	m<1	m<2	m<AVC
200°	40°	100°	20°	80°

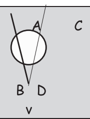


Theorem: The measure of an angle formed by two secants that intersect in the exterior of a circle is half the *difference* of the measures of the intercepted arcs.

3 1/2 minutes

Private Class Work: Example One

Given: m <AVC=60°, m AC=130°, find m $\widehat{BD}$



1 1/2 minutes

Public Class Work

A student puts the answer to example one on the overhead: (130°–x°)/2 = 60, 130°–x°=120°, x=10°, BD=10°

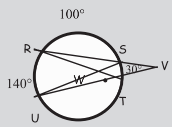
6 minutes

Private Class Work: Example Two

Given: m $\widehat{UR}$=140°, m $\widehat{RS}$=100°, m $\widehat{ST}$=30°,

Find m <RSU, m <RVU, m <USV, m <RWS

(W is the point inside the circle.)



3 1/2 minutes

Public Class Work

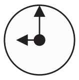
Students give answers to example two: m <RSU= 70°, m <RVU=55°, m <USV=110°, m <RWS=95°.

The teacher reminds students to study for their final exam tomorrow.

Figure 1.2 A typical Japanese lesson from the TIMSS video study.

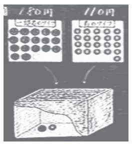
[54 Minute Lesson]

Japan Public Release Lesson 3 Lesson Graph [8th Grade]



4 Minutes

Public Class Work: Setting up the Problem– **The teacher reads the following problem to his class:**



It has been one month since Ichiro's mother entered the hospital. He has decided to give a prayer with his small brother at a local temple every morning so that she will be well soon. There are 18 ten-yen coins in Ichiro's wallet and just 22 five-yen coins in his smaller brother's wallet. They have decided every time to take one coin from each of them and put them in the offertory box and continue the prayer up until either wallet becomes empty. One day after they were done with their prayer, when they looked into each other's wallet the smaller brother's amount of money was bigger than Ichiro's. How many days has it been since they started the praying? That is the problem.

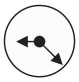
The teacher says, "I think that there are points hard to understand with just the sentences, so I would like to look at the figure and check it. He goes on to simulate the problem by taking coins from each wallet and putting them in the offertory box, asking how much in each wallet and which brother has more.

13 Minutes

Private Class Work: Students work individually on the problem

Midway through, the teacher says to the whole class, "If you found the answer with one method, try finding another method."

As he circulates around the room, he talks with several students about their solution methods and tells them that he is going to have them present their solution methods later on. He says to one student, "please think beforehand why you formulated an equation like this."



24 Minutes

Public Class Work: Students Presenting Solutions to Class

The teacher asks the students to present their solutions in the following order and place a pre-written title over each solution. He asks after each solution is shared, "How many others solved it in the same way?"

1. **Student one** (Manipulating actual objects)
Actually take one coin from each wallet and put it in offertory box until Ichiro's wallet contains less money than the brother's wallet. Or crossing out one coin from each wallet until the same condition is met. Answer: 15th day

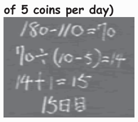
2. **Student two**
(Solving it by making a table)

Number of days	1	2	3	-----	14	15	16	17
Amount left	170	160	150	-----	40	30	20	10

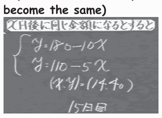
Number of days	1	2	3	-----	14	15	16	17
Amount left	105	100	95	-----	40	35	30	25

Answer: 15th day

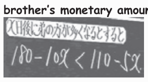
3. **Student three**
(There is a difference of 5 coins per day)



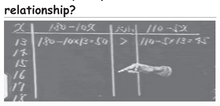
4. **Student four** **(If X is the day when the monetary amounts become the same)**



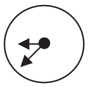
5. **Student five** **(If X is the day when the brother's monetary amount exceeds Ichiro's)**



Task: **Complete the chart and check if the value of x holds true for this inequality. What is the relationship?**



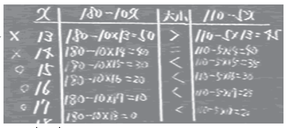
The teacher assigns a task and students work on it individually for six minutes.
The teacher asks a student to write her results on the chalkboard while the other students are still working.



13 Minutes

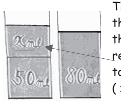
Public Class Work:

The teacher asks how many students had the same results as the last student. He notes that x holds true for 15, 16, 17 and 18; the first one was >. The second one was equal (14) which he calls "the standard".



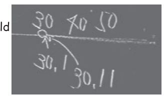
The teacher asks about the 19th day, and a student says Ichiro's wallet becomes empty and at that time they will end it. Students are asked to write the solution on their handouts.

The teacher presents a handout and reads the problem to the class:
The prayer was answered and their mother was able to leave the hospital safely, and that night they gave a toast with juice. At present there are 50 milliliters of juice in Ichiro's cup and 80 milliliters in the smaller brother's cup remaining. When the mother poured juice into Ichiro's cup it made Ichiro's cup reversely become more. How many milliliters had she poured?



The teacher asks the students to form an inequality to solve this problem. A student comes up to the board and says that the amount needed to get up to the 80 ml should be x ml (x represents the increased amount). The teacher asks students to write the situation with the unknown x ml and the symbols (> , <) . He says, "Using symbols like this, try expressing it."

After a minute, the teacher asks for the expression.
Student: 50 + x > 80. The teacher asks, "What values of x hold true for this equation?" Another student: "More than 30."
The teacher suggests using a number line and asks, "How many numbers are there more than 30?"
A student responds "Infinite".



The teacher asks if 30 is included or not included. When the student responds that it's not included, the teacher responds yes, but if it comes over even a little, like 30.1, it becomes more than.

The TIMSS video studies reveal somewhat of an irony: although the NCTM standards documents were written in North America, lessons in Japan are generally more aligned with NCTM standards than lessons in the United States are (Jacobs et al., 2006). The U.S. goal of "learning terms and practicing procedures" conflicts with many of the ideas (discussed earlier in this chapter) in NCTM's *Principles and Standards for School Mathematics.* One of the central messages of the document is that students must learn mathematics with understanding—it is not enough to learn terms and practice procedures; students must understand *why* the procedures work and *how* they are related to one another. On the other hand, the Japanese goal of "structured problem solving" closely fits NCTM goals. According to the problem solving process standard, students should construct new mathematical knowledge in the process of solving problems. By carefully sequencing problems to capitalize on students' prior learning, Japanese teachers work toward this goal.

Implementing the Common Core

See Homework Task 5 to view a video of TIMSS lessons online and analyze the extent to which the lessons help students "make sense of problems and persevere in solving them" (Standard for Mathematical Practice 1).

What conclusions are to be drawn from the differences in teaching patterns between the United States and Japan? If the U.S. instructional paradigm could switch overnight from "learning terms and practicing procedures" to "structured problem solving," would the United States "catch up" to Japan on achievement tests? Of course, it is impossible to answer that question because of a host of contextual factors unique to each country (Kaiser, Luna, & Huntley, 1999). Nonetheless, there are important things to learn from the Japanese lesson pattern. In particular, the Japanese pattern illustrates how it is possible to organize mathematics instruction around problem solving. Problem solving, in contrast to memorizing disconnected bits of information, is at the heart of doing mathematics. This chapter concludes with a discussion of what it means to teach through problem solving and how one might begin to do so.

STOP TO REFLECT

React to the statement "Problem solving, in contrast to memorizing disconnected bits of information, is at the heart of doing mathematics." Do you agree? Explain why or why not.

Teaching *Through* Problem Solving: The Centerpiece of Reform-Oriented Instruction

To understand what it means to teach through problem solving, it is helpful to first consider what it does *not* mean. Schroeder and Lester (1989) distinguished among **teaching *about* problem solving**, **teaching *for* problem solving**, and **teaching *through* problem solving**. Teaching *about* problem solving involves helping students learn general problem solving strategies. For example, some curricula introduce Polya's (1945) problem solving process with the assumption that students will adopt his thinking strategies as they solve problems of their own. Teaching *for* problem solving consists of explicitly teaching students mathematical ideas that they are expected to use to solve problems later on. This is closely aligned with the pattern of "learning rules and practicing procedures" that is widespread in the United States.

Implementing the Common Core

See Homework Task 6 to analyze the thinking processes students must use to "make sense of problems and persevere in solving them" (Standard for Mathematical Practice 1).

In contrast, teaching *through* problem solving involves selecting problems containing important mathematics and helping students learn mathematical ideas as they solve the problems. Teaching *through* problem solving aligns with the typical Japanese lesson pattern described earlier as "structured problem solving."

It would be inaccurate to assume that the approach of teaching *through* problem solving can produce positive learning results only for Japanese students. Studies of curricula developed in the United States that focus on teaching *through* problem solving show that students experiencing the curricula perform as well as their counterparts from traditional classrooms on conventional tests of content, while generally outperforming them in problem solving, reasoning, and nontraditional content (Chappell, 2003; Putnam, 2003; Swafford, 2003). Therefore, teaching through problem solving is not an abstract, unrealistic, idealized notion. It is supported by careful studies of teachers and students, unlike more traditional forms of instruction that are widespread in the United States. As you prepare to teach *through* problem solving, it is important to realize that it requires a great deal of work. It is much more challenging than simply having students copy notes from a board or a screen.

Teaching through problem solving begins with the selection or design of student tasks. NCTM (1991) provides the following thoughts about **worthwhile mathematical tasks**:

> Good tasks are ones that do not separate mathematical thinking from mathematical concepts or skills, that capture students' curiosity, and that invite them to speculate and to pursue their hunches. Many such tasks can be approached in more than one interesting and legitimate way; some have more than one reasonable solution. (p. 25)

After making this statement, NCTM provided an example of one task not likely to fit the description and another that was likely to fit (Figure 1.3).

Figure 1.3 Comparison of two tasks involving area and perimeter (NCTM, 1991, p. 28).

TASK 1:

Find the area and perimeter of each rectangle:

TASK 2:

Suppose you had 64 meters of fence with which you were going to build a pen for your large dog, Bones. What are some different pens you can make if you use all the fencing? What is the pen with the least play space? What is the biggest pen you can make—the one that allows Bones the most play space? Which would be best for running?

There are several important differences between Tasks 1 and 2 in Figure 1.3. Task 1 simply requires the skill of calculating area and perimeter. Task 2, in contrast, provides the opportunity to build understanding of area and perimeter as students solve a problem involving both concepts. Task 2 can be tackled in a variety of ways, ranging from trial and error to using tools from algebra and calculus. Task 2 can also be readily extended—for example, after solving the given problem successfully, students could be asked to make generalizations about which types of pen arrangements provide the most area.

Selecting and posing a problem does not mark the end of the teacher's role in the process of teaching *through* problem solving. Contrary to what some believe about this approach, the teacher does not just sit back passively as students solve problems. Instead, the teacher plays an active role in facilitating students' learning. For instance, the teacher can help students organize their thinking by having them make initial conjectures about what the answer to the problem might be (Cohen & Adams, 2004). As students work through the problem, they can then judge the reasonableness of their solution strategies by referring back to their initial predictions.

Implementing the Common Core

See Homework Task 7 to think about modifying exercises in traditional textbooks to make them "worthwhile mathematical tasks" that can set the stage for students to "make sense of problems and persevere in solving them" (Standard for Mathematical Practice 1).

Teachers can also help draw students' attention to the structure of a problem by asking them to draw diagrams that represent the key quantities and having them "act out" the problem when possible. For example, suppose students were asked to solve the following problem:

> Fran ate one-half of a pizza. Tom ate three-fourths of another pizza that was the same size. They decided to combine the amount of pizza they had left over. What fraction of a pizza would they have if they did so?

To draw students' attention to the quantities in the problem, one could ask them to make drawings to show the relevant quantities of pizza. They might also use manipulatives such as fraction circle pieces to act out the manner in which the quantities of pizza change over time and are eventually combined. This sort of **quantitative analysis of a task** leads to much deeper learning than the traditional "key word" approach students are sometimes asked to follow (L. L. Clement & Bernhard, 2005). Notice, for instance, that students would not have a chance of being successful with the fraction problem above if they simply relied on a rule such as "The word *of* means multiply."

STOP TO REFLECT

Imagine that you are a student who has not yet learned any of the rules for adding, subtracting, multiplying, or dividing fractions. However, you are familiar with how to represent fractions using diagrams and manipulatives. Write a solution to the Fran and Tom pizza task above from this perspective.

As teachers use various strategies to facilitate problem solving, they need to be careful not to give away so much that the challenge of the task is taken away. Stein, Grover, and Henningsen (1996) described how teachers sometimes do this during problem-based lessons. They observed that as students press the teacher to provide solutions to problems, the teacher might show students a procedure or set of steps that will yield

Implementing the Common Core

See Homework Task 8 to reflect on how the challenge of a task is related to students' tendency to "make sense of problems and persevere in solving them" (Standard for Mathematical Practice 1).

the solution to the problem. For example, in regard to the fraction problem above involving pizzas, a teacher might reduce the challenge of the problem by telling students to find a common denominator for one-half and one-fourth and add the two fractions to obtain the solution. Notice that in doing this the teacher takes away students' opportunity to represent the quantities in the problem and think about the action in it. Although the teacher may do so with good intentions, it is important to note that what has happened, in essence, is that the lesson has reverted from "structured problem solving" to "learning rules and practicing procedures." Care must be taken to avoid this trap.

First Steps in Beginning to Teach Through Problem Solving

Since teaching through problem solving is so central to standards-based teaching, subsequent chapters of this textbook will have much more to say about it. For now, the discussion of this topic will conclude by addressing two of the concerns teachers often have as they begin to think about teaching through problem solving. The first involves curricular resources, and the second involves time constraints.

Teachers often wonder where they will find the curricular resources to teach through problem solving. The answer to that question is influenced largely by one's school setting. Most schools have adopted textbooks that *claim* to support NCTM standards–based instruction or the *Common Core State Standards*. It is actually quite difficult to find a mathematics textbook on the market that does not make such a claim. However, many of these texts (with the notable exceptions of some of the curricula that are discussed in Chapters 4 and 5) are designed with the assumption that classrooms are set up to follow the traditional pattern of "learning rules and practicing procedures." In this case, teachers may have to modify exercises in the book to transform them into worthwhile mathematics tasks. It is also helpful to draw tasks from teachers' journals such as *Mathematics Teaching in the Middle School* and *Mathematics Teacher*. Lesson plans in Chapters 7 through 11 of this book model how one can use articles from these journals to construct lessons.

Another worry about teaching through problem solving is that it will take too much instructional time. This is an especially pressing concern in light of schools' desire to prepare students for high-stakes standardized tests. Keep in mind, however, the research evidence showing that students who experience a problem-centered curriculum generally perform as well as or better than students from traditional settings on tests of computation and higher-level thinking. It is usually more inefficient, in terms of enhancing students' achievement, to follow the pattern of learning rules and practicing procedures. If students who are used to simply learning rules and practicing procedures encounter a problem on a test for which they have forgotten the procedure, they tend to shut down, but those who have experienced teaching through problem solving have generally developed the disposition to reason through the problem even if it looks unfamiliar (Boaler, 1998). While carefully "covering" the curriculum in discrete chunks by having students learn rules and practice procedures may seem to be a commonsensical, safe way to teach, research does not support its effectiveness. Subsequent chapters of this book offer research-supported alternative approaches.

CONCLUSION

Several conclusions can be drawn from this chapter. First of all, it is an awesome responsibility to teach mathematics and a great challenge to teach it well. Anyone can teach mathematics, but it takes skill to teach it well. Good teachers seek out teaching strategies that will most effectively support their students' learning. This book introduces some of those strategies. However, it is ultimately the teacher's responsibility to study students' needs carefully to begin to understand how to apply and adapt those strategies to best facilitate learning. While some believe that the mathematics teacher's job is done once a set of notes has been transcribed onto an overhead projector or chalkboard, NCTM standards and the TIMSS studies point out that *effective* mathematics teaching is much more complex. The purpose of this textbook is to help teachers begin to understand and navigate some of those complexities, and also to help them develop the thinking skills to continue to do so throughout their teaching careers.

VOCABULARY LIST

After reading this chapter, you should be able to offer reasonable definitions for the following ideas (listed in their order of first occurrence) and describe their relevance to teaching mathematics:

National Council of Teachers of Mathematics 3

Curriculum and Evaluation Standards for School Mathematics 3

Professional Standards for Teaching Mathematics 4

Assessment Standards for School Mathematics 4

Principles and Standards for School Mathematics 4

NCTM principles 6

NCTM process standards 6

Common Core State Standards 10

Third International Mathematics and Science Study (TIMSS) 12

Teaching about problem solving 15

Teaching for problem solving 15

Teaching through problem solving 15

Worthwhile mathematical tasks 16

Quantitative analysis of a task 17

HOMEWORK TASKS

1. NCTM's *Curriculum and Evaluation Standards* called for more attention to "the use of real-world problems to motivate and apply theory" and less attention to "word problems by type, such as coin, digit, and work." Give an example of how word problems might be used for each of these purposes and then explain the difference. Draw on the *Curriculum and Evaluation Standards* as necessary in writing your response.
2. Search for the phrase "shaping the standards" on the www.nctm.org search engine (put the search phrase in quotes). Using your search results, describe how NCTM sought to have teachers contribute insights to the *Principles and Standards for School Mathematics* as they were written. Also describe

how teachers' input actually helped shape NCTM standards. Note: You will need access to NCTM's journals, either online or through your library, for this activity.

3. Visit the websites of two different sides of the "math wars": www.mathematicallycorrect.com and www.mathematicallysane.com. What is the stated purpose of each website? How well does the stated purpose align with the material posted? Provide specific examples to support your assertions.

4. Visit the TIMSS website at http://nces.ed.gov/timss/Educators.asp to see some of the items included on international assessments. Pick three items on which the United States scored poorly. For each item, form a conjecture as to why students may have scored poorly. Then, discuss how following specific aspects of the NCTM standards documents might help improve students' achievement on the problems you identified.

5. Visit the TIMSS websites http://nces.ed.gov/timss and http://timssvideo.com/ to view sample lesson video clips from the United States, Japan, and other countries. Which lessons seem to reflect the goal of "learning terms and practicing procedures"? Why? Which lessons seem to reflect the goal of "structured problem solving"? Why?

6. Find three sources that discuss the components of George Polya's problem solving process. Then describe a situation in which you solved a mathematics problem for one of your undergraduate mathematics classes. As you describe the situation, explain how the process you used to solve the problem did or did not align with Polya's ideas.

7. Find a traditional textbook or curriculum series that takes the approach of teaching *for* problem solving. Select one of the problems and rewrite it to align more closely with the guidelines for a "worthwhile mathematical task" from NCTM's (1989) *Professional Standards for Teaching Mathematics* (see pp. 25–32 in that document). Draw on the following article to stimulate your thinking, and in your write-up, explain how it helped you:

   Kabiri, M. S., & Smith, N. L. (2003). Turning traditional textbook problems into open-ended problems. *Mathematics Teaching in the Middle School, 9,* 186–192.

8. Read the following article from the February 1998 issue of *Mathematics Teaching in the Middle School:*

   Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3,* 344–350.

After reading the article, reflect on your own experiences in learning mathematics in Grades 6 through 12. Did your teachers ask you to function at high or low levels of cognitive demand? Explain.

CLINICAL TASK

Observe a full-length mathematics lesson. As you observe the lesson, put an X in the appropriate place in the following chart indicating how well the lesson is aligned with the given standard, then write an explanation justifying each rating. You should read through the description of each of the standards in Appendix A before completing this assignment. Revisit and refine your ratings and explanations as you read the rest of the chapters in this book.

	High Alignment	Some Alignment	No Alignment	Explanation
CCSS Standards for Mathematical Practice				
1. Make sense of problems and persevere in solving them				
2. Reason abstractly and quantitatively				
3. Construct viable arguments and critique the reasoning of others				
4. Model with mathematics				
5. Use appropriate tools strategically				
6. Attend to precision				
7. Look for and make use of structure				
8. Look for and express regularity in repeated reasoning				

Use the chart to write a paper that summarizes the degree of alignment observed between the lesson and each of the indicated standards. Submit your final, polished paper at the end of the course.

VIGNETTE ANALYSIS ACTIVITY

Focus on Making Sense of Problems and Persevering in Solving Them (CCSS Standard for Mathematical Practice 1)

Items to Consider Before Reading the Vignette

Read CCSS Standard for Mathematical Practice 1 in Appendix A. Then respond to the following items:

1. Drawing on your own experiences as a mathematics student, provide specific examples of teaching practices that can help students make sense of problems and persevere in solving them.
2. Drawing on your own experiences as a mathematics student, provide specific examples of teaching practices that might prevent students from making sense of problems and persevering in solving them.

Scenario

Ms. Horton was just beginning her student teaching experience. Before assuming the role of lead classroom teacher, she observed that her mentor teacher, Mr. Sanchez, had a very good relationship with his students. Ms. Horton worried that her students might not be as accepting of her as Mr. Sanchez's students were of him and wondered how she could establish the same sort of productive relationship with her group. To help motivate students to work for her, Ms. Horton decided it would be a good idea to set the problems she posed in interesting real-world contexts. In the seventh-grade lesson described below, her goal was to help students understand the similarities and differences between circumference and diameter. She also wanted students to understand the number π as the ratio of circumference to diameter in any given circle.

The Lesson

As students entered the room, Ms. Horton had three expressions on the board that they were to simplify for a warm-up activity: (i) $7 + 3(6) \div 2$; (ii) $\frac{11 \times 8}{32 \div (5 \quad 1)}$; (iii) $\frac{2 + 3 \times 14}{17 \quad 11}$. She set a timer for five minutes and projected it on the document camera to show students how much time they had to work. Most students settled in and worked on the problems individually. Ms. Horton noticed that a few students were not done when the timer rang, so she reset it for another two minutes. The students who needed extra time kept working, while those who believed they were done began to socialize with their neighbors.

When the timer went off again, she called three students to the board to put their work up for everyone to see. As the three students put their work on the board, the others continued to fidget in their seats and talk with one another. When all three problems had been worked at the board, Ms. Horton directed everyone's attention up front. She stated that the first two were correct but that there was a problem with the third. The student assigned to post the work for the third item added 2 + 3 to get 5 and then multiplied 5 by 14. Ms. Horton asked if students agreed that this was the correct way to simplify the problem. One student, Jamie, spoke up, saying that the multiplication in the numerator should have been done before the addition. Ms. Horton told Jamie she was correct, because multiplication comes before addition in order of operations.

The lesson then moved on to a discussion of the concepts of circumference and perimeter. Ms. Horton asked students to give examples of things that have circumference. Students quickly blurted out several examples, such as "coffee cup," "ring," "ball," "lightbulb," and "doorknob." Ms. Horton took this as enough evidence that students understood the idea of circumference and moved on to discuss how one would determine the perimeter of a rectangle. She explained that finding the perimeter of a rectangle was much like determining circumference because both involved calculating the distance around the outside of an object.

Following the discussion of circumference and perimeter, Ms. Horton announced that the students' first main problem for the day would be to determine the perimeter of a rectangular cookie using a ruler. She called on three students to distribute rulers and another three to distribute cookies. Materials were distributed efficiently and quickly. Students began to use the rulers to measure around the outside of the cookie. Most finished this task quickly and were allowed to eat their cookie. While eating, many of them talked with one another about the football playoff game the previous night. A group of students near the back of the room had difficulty reading the rulers and adding up the fractional portions of centimeters for each side length, so a teacher's assistant helped them finish the task.

Ms. Horton then moved on to what she considered to be the most challenging problem of the day. She held up a circular Oreo cookie and asked students if they could determine its perimeter. She then took out a box of Oreo cookies and had three students distribute them to the rest of the class. Students were told to use their rulers to try to determine the distance around the outside. Some became frustrated and said it was impossible to do the task using a ruler. The teacher's assistant told some of the students who became frustrated to wait a few minutes, because the teacher would be handing out string to help them measure around the outside. Other students began to roll the cookie along the ruler to measure it. Before anyone could formulate a response, Ms. Horton announced, "OK, you can see that it is really hard to do this with just a ruler. So, now I will give you a piece of string to use as well." After the string was distributed, Ms. Horton circulated about the room to direct some students to wrap the string around the cookie and then place the string on the ruler to measure the length used. Many students finished the task early and then resumed their football conversations. A few needed help measuring precisely, so Ms. Horton and the teacher's assistant helped them finish while the others waited.

After the class had finished measuring the circumference of the Oreos, Ms. Horton asked students to find other circular objects in the room and measure and record their circumferences and diameters. As students started to do so, Ms. Horton noticed that some of them were measuring the radius rather than the diameter of the circular objects. Others were measuring chords that did not pass through the center of the circle. On seeing this, Ms. Horton drew the attention of the class back to the front of the room and demonstrated how to measure the diameter of a circle.

When students had finished measuring the circumference and diameter of several circular objects, Ms. Horton once again called their attention up front. She asked students how many diameters fit into a circumference for each circular object. One student, Jessica, stated, "You need three diameters and a little bit more." Ms. Horton told Jessica she was correct, and said that the "little bit more" was the 0.14 in the 3.14 number they had been using for π.

To finish the lesson, Ms. Horton had students copy some notes from the document camera. The notes included formulas for determining the perimeter of a rectangle and the circumference of a circle. Then, Ms. Horton put a calculator on the screen and typed in $22 \div 7$. The calculator produced the decimal 3.14285714. She told students to notice that 3.14 was the number they were using for π in their circumference formula, and said it would be fine to use either 3.14 or $\frac{22}{7}$ for π when doing their homework. Students were told to write $\frac{22}{7} = \pi$ in their notebooks as a reminder.

Questions for Reflection and Discussion

1. Which aspects of the CCSS Standard for Mathematical Practice 1 did Ms. Horton's students seem to attain? Explain.
2. Which aspects of the CCSS Standard for Mathematical Practice 1 did Ms. Horton's students not seem to attain? Explain.
3. Comment on the overall value and relevance of Ms. Horton's warm-up activity to her objectives for the day.
4. Critique Ms. Horton's time management during the lesson. Identify instances of downtime and propose strategies for eliminating them.
5. How could Ms. Horton's problems for the day be enriched to offer extra challenges while still being accessible to all students?
6. There was a teacher's assistant in the classroom. How might this individual be used to effectively support the implementation of the lesson?
7. Did Ms. Horton provide appropriate support for students' problem solving activities during the lesson? Were there instances in which more support was necessary? Were there instances in which too much guidance was given? Explain.
8. Comment on the appropriateness and correctness of the statement $\frac{22}{7} = \pi$.

RESOURCES TO EXPLORE

Books

Burke, M. J., Hodgson, T., Kehle, P., & Resek, D. (2006). *Navigating through mathematical connections in Grades 9–12.* Reston, VA: NCTM.

Description: Connections can be made through a variety of different means in mathematics classes. This book provides classroom activities that build connections through mathematical models, unifying themes, multiple representations, and problem-solving processes.

Burke, M. J., Luebeck, J., Martin, T. S., McCrone, S. M., Piccolino, A. V., & Riley, K. J. (2008). *Navigating through reasoning and proof in Grades 9–12.* Reston, VA: NCTM.

Description: This book emphasizes exploration, conjecture, and justification as key to enacting the reasoning and proof process standard in high school classrooms. Reasoning and proof classroom activities related to each of the five NCTM content standards are included.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM. Available online at http://standards.nctm.org

Description: This document has been highly influential in framing debates about mathematics education and the scope of the preK–12 curriculum. Its influence can be seen across past and present state-level curriculum documents.

Pugalee, D. K., Arbaugh, F., Bay-Williams, J. M., Farrell, A., Mathews, S., & Royster, D. (2008). *Navigating through mathematical connections in Grades 6–8.* Reston, VA: NCTM.

Description: Two types of connections are emphasized in the NCTM connections process standard: connections within mathematics and connections between mathematics and other disciplines. This book contains classroom activities suitable for helping middle school students forge both types of connections.

Thompson, D. R., Battista, M. T., Mayberry, S., Yeatts, K. L., & Zawojewski, J. S. (2009). *Navigating through problem solving and reasoning in Grade 6.* Reston, VA: NCTM.

Description: This book contains examples of how teachers may enact NCTM's vision for the process standards of problem solving and reasoning in the middle grades. Specific activities involve understanding area formulas, working with scale factors, and reasoning about data.

Websites

Common Core State Standards: **http://www.corestandards.org/**

Description: Written by a consortium of 48 states, the *Common Core State Standards* represent the closest we have come to a consensus on the mathematics to be studied by students in the United States. This website contains the full text of the standards and news related to their implementation.

Official Website of TIMSS Public Use Videos: **http://timssvideo.com/**

Description: The site contains videos of mathematics lessons from seven different countries participating in the TIMSS video studies. The videos help illustrate some of the typical features of mathematics instruction in each country.

Teaching Math: A Video Library 5–8: **http://www.learner.org/resources/series33.html**

Description: These videos show middle school teachers implementing the recommendations in the NCTM standards in the classroom with their students. Lesson topics include fractions, statistics, geometry, measurement, and functions.

Teaching Math: A Video Library 9–12: **http://www.learner.org/resources/series34.html**

Description: These videos show high school teachers implementing the recommendations in the NCTM standards in the classroom with their students. Lesson topics include functions, mathematical modeling, linear programming, and probability.