

# Introduction

*The illiterate\* of the 21st century will not be those who cannot read and write, but those who cannot learn, unlearn, and relearn.*

Alvin Toffler

Everywhere we turn these days we encounter another article, report, or book on the importance of algebra. In 2000, the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM]) made algebra one of the five mathematics content standards for preK–12 mathematics. NCTM describes algebra as a way of thinking that cuts across all math content areas and unifies the curriculum. Yet 12 years after this major publication, many national and international reports,<sup>1</sup> in acknowledging a serious national mathematics problem, consistently identify algebra as a central concern. Why? Because the mathematics achievement curve begins a sharp decline as students reach late middle school, precisely when U.S. students begin their study of algebra. Classic comments such as “I liked math until I began Algebra” or “Math made sense until Algebra” add anecdotal evidence to scientific results. *Note:* The word *algebra* in this book is capitalized only when referring to a particular course, such as Algebra I or Algebra II.

To help remedy this situation, many reports on the state of mathematics in the United States are calling for a strong foundation for algebra in the early grades in order to prepare students for success in high school Algebra. But questions arise about how best to answer this call. After all, elementary teachers were not educated to teach algebra, and, what’s more, they may barely remember high school algebra. Some may even harbor unpleasant memories of their algebra experience. So, are elementary teachers being asked to teach those formal symbolic manipulations with  $x$ ’s,  $y$ ’s, and  $z$ ’s to young children? The answer is an emphatic NO!

Algebra in the early years, a relatively new focus area in mathematics education, has received much attention in the past decade. This movement, known as “early algebra” to the math education community, is *not* about teaching traditional school algebra *early*. Rather, it’s about fostering ways of thinking about, doing, and communicating about mathematics, and of teaching and learning mathematics with understanding. It’s about making connections, analyzing relationships, noticing structure, studying change, and solving problems; it’s about justifying, conjecturing, generalizing, symbolizing, and mathematizing, all of which are critical habits of mind.

But this begs further questions: How can elementary teachers learn to cultivate these habits of mind? Where can they find the research on children’s algebraic thinking in

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\*Applies to the innumerate as well [author’s addition].



**“Algebra will be useful to you later in life because it teaches you to shut up and accept things that seem pointless and stupid.”**

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To be more specific, I have written this book for teachers in the early grades, whose students are years away from encountering Algebra as a formal course, to help instill in them a very different view of algebra than the popular one captured in the Glasbergen cartoon above! It is my hope that readers of this book will

- *Unlearn* any negative lessons about algebra they may be harboring and *release* any negative beliefs or attitudes that impede a full appreciation of the topic
- *Experience* algebraic acculturation: that is, *cultivate* new thought and behavior patterns that naturally weave into algebra’s cognitive fabric
- *Reconceptualize* algebra as a domain that makes sense and connects to the world we live in, rather than as a set of meaningless symbolic procedures
- *Understand* the continuum—and visualize meaningful bridges—between the mathematics they teach and the mathematics taught in secondary school algebra
- *Visualize* concrete embodiments of what algebra actually looks like in the early grades, giving it color, light, and texture
- *Enjoy* the usefulness, power, and beauty of algebra as an integral part of mathematics

## Why Write This Book?

Any author who writes a book for teachers does so in the hope and anticipation that certain educational outcomes will follow, and I am no exception. As I finish this book about the hidden algebra connections in the mathematics curriculum for our youngest students, and begin work on the next volume for teachers of students in grades 3–5, it seems important to articulate the hoped-for outcomes that are the motivating force behind this work. Chapters 1 through 10 are primarily involved at the micro level with interactions in the

forms clear and useful to them? In what ways could they use their present curriculum to meet their algebra expectations? In short, what does it *mean* to plant the seeds of algebra early, and what do these seeds *look like* in grades preK through 2? One answer is in your hands. *Planting the Seeds of Algebra, PreK-2: Explorations for the Early Grades (Planting Seeds)* is a pioneer in its genre: It presents engaging content, models teaching strategies, connects the early mathematics with advanced algebra concepts, and makes suggestions for further explorations. Based on existing research, these different components converge to offer meaningful algebraic experiences for young students.

classroom and the small though significant understandings that take place during a mathematics lesson. What follows here are the larger, macro-level consequences of those small understandings, as I envision them: the positive, broad-ranging results to be gained as a result of the cumulative incremental shifts in mathematical instruction that routinely “plant the seeds” of algebraic thinking.

**Student Achievement and Teacher Empowerment.** As teachers begin to “algebrafy” (term introduced by Early Algebra pioneer James J. Kaput) their elementary mathematics by weaving into it a web of algebraic ideas and actions, words and symbols, their students will become better at doing mathematics, and together they will enjoy the subject more. Teachers will be able to answer rather than dread those *why*-questions such as, “Why *couldn't* I subtract 8 from 5?” Moreover, since algebra connects to all areas of mathematics, students will also gain better number sense, spatial sense, and symbol sense. When algebra is regarded as an organizing principle for elementary mathematics, the potential for increased math proficiency is huge. Mindful explanations engender profound understanding, which in turn contributes to richer mathematical experiences in early grades, which finally lead to pleasurable and successful mathematics experiences in high school and college. For these reasons, I anticipate teachers will feel empowered after reading and using *Planting Seeds*.

Mindful explanations → Profound understanding → Richer experiences → Pleasure and success

**Respect for Elementary Teachers.** Throughout my 25 years of professional development with preK–14 teachers, I’ve heard high school teachers blame elementary teachers too often for their students’ algebra ills. Granted, we have a national math problem, and algebra is clearly at center stage. But good will is not the missing ingredient: I’ve worked with many reflective, inquisitive, and assiduous elementary school teachers who are open-minded and eager to learn.

The reason for middle and high school students’ poor foundation for Algebra is at least twofold: (1) Elementary teacher preparation programs require little by way of mathematics, and (2) the elementary teachers who venture to take an Algebra course are exposed to a traditional high school or college algebra course, neither of which teaches them to foster algebraic reasoning and thinking in their elementary students. Therefore this book meets an urgent need: Its content will inspire teachers to create a new classroom culture of algebraic approaches to mathematics, the fruits of which will have an upward domino effect, forcing secondary math teachers, in turn, to rethink their own algebra teaching practices. As a result of this inevitable change, high school teachers will stop trivializing the mathematics learned and taught in elementary school and begin valuing elementary teachers.

LOWER SCHOOL MATH. The indoctrination begins. Students learn that mathematics is not something you do. . . . Emphasis is placed on sitting still, filling out worksheets, and following directions. Children are expected to master a complex set of algorithms . . . unrelated to any real desire or curiosity on their part. . . . Multiplication tables are stressed, as are parents, teachers, and the kids themselves.

Paul Lockhart (2009)

**A Mathematics Revolution With Teachers as Agents of Change.** There is a dire need to educate the public about algebra in particular and mathematics in general. To change beliefs, attitudes, and behaviors. To reintroduce these noble intellectual achievements back into our culture as valuable assets. This can only happen through education. And this education must start with the very young. Elementary school teachers can be change agents in this burgeoning revolution: They are educating the leaders and decision makers of tomorrow. Only they can model for our children a mathematics that is vibrant, useful, exciting, and rich. Hence, this book will plant the seeds of a new awareness of the nature of algebra (and mathematics as a whole) that will begin to change current attitudes. It will spread from teachers to students and outward to society. A new awareness about the role of algebra in our world will gradually deviate from opinion (1) below, and align itself more with opinion (2) below.

(1) Algebra isn't essential to much of anything. . . . It is useless torture. . . . It's for the few, not the many.

Colman McCarthy (1991)

(2) Algebra represents one of mankind's great intellectual achievements—the use of symbols to capture abstractions and generalizations, and to provide analytic power over a wide range of situations, both pure and applied.

Alan H. Schoenfeld (2008)

**The Democratization of Algebra.** High school Algebra has generally been acknowledged to serve as a gatekeeper in education: It lets some people in to enjoy rewarding STEM (science, technology, engineering, and/or mathematics) careers but keeps many others out. I have been passionate about the need to widen algebra's narrow gate since the inception of my mathematics career. But still today, too few students pass Algebra with success and understanding. Since algebra continues to be considered a criterion for success in higher education, failure in high school Algebra disadvantages a slice of the population that includes many female, Black, and Hispanic students, thus depriving them of careers in science or fields that require mathematics proficiency. Recent reports have further correlated success in higher education institutions with completion of Algebra II. Reconceptualizing the nature of algebra in the early grades will provide *all* students opportunities to engage with the big ideas of algebra in meaningful ways and consequently increase the chances for more students to complete high school Algebra I and II. This book will contribute to the early algebra reform efforts begun by others to transform Algebra from "gateway for some to highway for all" (Neagoy, 2010).

Algebrafying the K-12 curriculum will transform Algebra from an engine of inequity to an engine of mathematical power.

James J. Kaput (2008)

**Narrowing of the Global Achievement Gap.** Beyond college and graduate school, we must also consider the quality of work and life itself. A growing number of educational scholars and leaders are concerned about our students' disadvantages on a larger scale. Harvard education professor Tony Wagner warns of a global achievement gap (Wagner, 2008). He argues that the 2001 No Child Left Behind Act, instead of narrowing the achievement gap, has left us with ineffective schools unable to prepare our students for college, work, and life. With judicious reasoning, he makes the case that our students are unprepared to analyze arguments, weigh evidence, or detect bias. He articulates a list of "seven survival skills" for today's teenagers, the core competencies he deems necessary for success in college and the workplace. Wagner's survival skills are strikingly similar to the habits of mind modeled in engaging mathematics classrooms, where mathematics is taught and learned with understanding. These include the following:

- Critical thinking and problem solving
- Agility and adaptability
- Effective oral and written communication
- Curiosity and imagination

*Planting Seeds* focuses on rethinking algebra as a network of ideas, actions, and symbols, and on fostering mathematical habits of mind that will serve students all the way through high school and beyond. These ways of thinking and doing will transfer to other areas of education and life and will thus be foundational to developing Wagner's survival skills.

One's intellectual and aesthetic life cannot be complete unless it includes an appreciation of the power and the beauty of mathematics.

Jerry P. King (1992)

**A Love for Mathematics Akin to the Love for Language Arts.** When it comes to mathematics, and especially algebra, we almost always justify its importance to students by portraying it as a prerequisite to (1) high school graduation, (2) college achievement, or a (3) lucrative career. In my opinion, a main ingredient is missing in this utilitarian portrayal of "one of humanity's most ancient and noble intellectual traditions" (RAND Mathematics Study Panel, 2003). The missing ingredient is a love—dare I say a passion—for mathematics, one akin to the delight in reading or the joy of creative art. A deep appreciation for the intrinsic nature of mathematics—its art, elegance, and creativity—is lacking. The many "wow moments" and "aha moments" teachers and students will experience together, using explorations from this book, will begin to foster a profound enjoyment of algebra and consequently of mathematics, and will lead, I hope, to a contagious "love" for its awesome beauty.

I wish to emphasize the critical role elementary school teachers play in kindling children's love for mathematics. Their effect not only on students' achievement in mathematics but also on students' love for mathematics is critical. It may be hard for research to prove it, but anyone who was inspired in the early grades knows the vital role his or her teacher played. Once students enter middle school, their minds are pretty much made up

about mathematics. For the disheartened, there's a slim chance of any rekindling. Therefore, we must reach children while they are young and *turn on* their minds—and *hearts*—to mathematics. A child *turned off* to mathematics is a tragedy for the child and a tragedy for our country.

## What Background Do I Bring to This Book?

First, *Planting Seeds* is the outgrowth of 25 years of working with teachers. I have worked specifically on algebra in three principal contexts: (a) teaching site-based professional development courses or workshops for teachers of all grade bands; (b) designing, directing, and teaching summer institutes for K–14 teachers at venues such as Georgetown University and the Carnegie Institution of Washington; and (c) training elementary teachers to become math specialists for their schools (e.g., the Alexandria City Public Schools Math Specialist Program).

Second, this book is based on the research knowledge I have acquired throughout my career through both doing and consuming research, and especially through guiding large-scale national algebra related projects as program director at the National Science Foundation under the umbrellas of the Teacher Professional Continuum (TPC) and the Centers for Learning and Teaching (CLT) programs.

Third, this book is also the fruit of my work over the past 14 years in creating, writing, and hosting television programs (e.g., on the Annenberg CPB Channel<sup>2</sup>) and video series (e.g., for Annenberg Media, Discovery Education,<sup>3</sup> Media4Math,<sup>4</sup> and T3 Europe<sup>5</sup>) on the teaching and learning of algebra in grades K–14.

Lastly, this book draws on presentations, keynote speeches, and Math Show performances I've given on the topic of algebra at national and international conferences, cultural events, and back-to-school or Math Nights for parents. I am grateful to the many people who insisted that I "write a book on algebra that would inspire more people."

## Who Will Benefit From This Book?

On a pragmatic level, I've written *Planting Seeds* primarily for elementary school teachers and for teacher educators, professional development providers, and teacher leaders who work with preservice or in-service elementary school teachers. Since more and more parents today are homeschooling their children, this book will also serve these parents as well as anyone else trying to make sense of the rapidly shifting K–12 algebraic landscape. On a philosophical level, this book joins my other lifelong professional efforts to inspire teachers, students, and all lovers of learning by *infusing*, *infecting*, and *injecting* them with a fascination for the power, the value, and the beauty of mathematics in general, and of algebra in particular.

I envision this book being used as a resource for the following:

- Workshops, courses, and institutes for in-service teachers and courses for preservice teacher preparation
- Site-based lesson study or other teacher collaborations
- Sessions for primary teachers run by math specialists and other teacher leaders
- Math workshops for parents of young children
- Individual learning by teachers, administrators, and other readers interested in children's mathematical development and education



I look forward to the appearance of other publications that will join this one in helping to inspire teachers and parents to turn children *on* to algebra early in their lives.

## How This Book Is Organized

Three in-depth Explorations—*Making 7*, *Sticker Stickler*, and *Patterns of Cubes*—are at the heart of this book. Explorations I, II, and III, which present material for preK–2 students on addition, subtraction, patterns, and special numbers, are in turn each composed of three chapters:

1. The Lesson (Chapters 1, 4, and 7)
2. Algebra Connections (Chapters 2, 5, and 8)
3. More Problems to Explore (Chapters 3, 6, and 9)

For each Exploration, *The Lesson* chapter recounts an actual primary lesson that was recorded in a primary classroom. (Students' comments are authentic, but their names have been changed.) *The Lesson* ends with a *Next Steps* section, which offers suggestions for follow-up lessons. The *Algebra Connections* chapter revisits each stage of the exploratory lesson, bringing to light the hidden connections between primary mathematics and high school algebra. Finally, the *More Problems to Explore* chapter, the last one in each of Explorations I, II, and III, offers three further explorations for the teacher's own learning and ten further explorations for the students.

Exploration IV, in Chapter 10, walks through a K–1 lesson I have often used with parents at events such as back-to-school nights, Math Nights, or “parent university” sessions I've instituted in some client schools. This lesson, *Double Deckers*, powerfully models what it means to treat elementary mathematics in algebraic ways.

### Grade Band

Explorations I through IV are labeled for grades preK–K, 1–2, 2, and K–1, respectively. Nevertheless, with small modifications, all explorations can be used in any primary grade (preK through 2). Moreover, *all* elementary school teachers will find the wide range of algebraic connections and additional problems relevant to the mathematics they teach.

### Exploration Sources

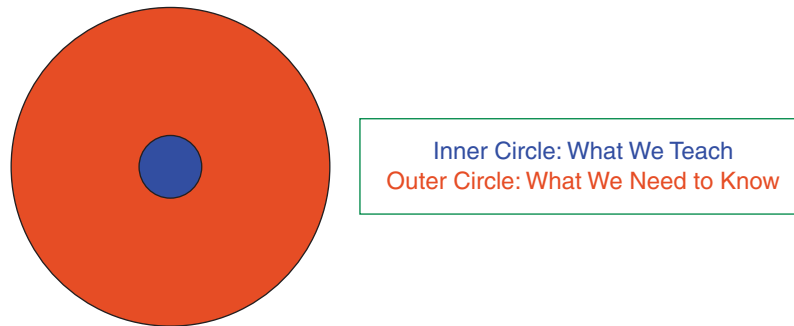
The particular lessons in this book come from “model teaching” sessions with the students of classroom teachers I work with in public and private schools. In terms of topics, Explorations I and II focus on operations (primarily, addition and subtraction) as relationships, and their properties; Exploration III focuses on the not-so-obvious connections between patterns and functions; and Exploration IV examines the nature and properties of odd and even numbers and the relationship between them. As NCTM (2011a) validates, these topics are central to early algebra: “The role of algebra in primary grades should go far beyond occasional appearances in repeating and growing patterns. In fact, to switch metaphors, we might describe algebra as a lens for magnifying relationships and properties of number and operations”

## My Premises

As you read through and engage in this book's explorations, you will no doubt notice the following premises that have held true throughout my career.

**Premise 1: We must be motivated and desirous to engage with mathematics.** When mathematics is explored in ways that reveal its usefulness, its beauty, and its humanity, teachers and students alike develop a motivation and desire to engage with it on emotional and intellectual levels. Engagement at both these levels is a critical ingredient for a positive and rewarding mathematical experience.

**Premise 2: We need to know much more than what we teach.** Teachers are professionals. They understand that they must know much more than what they teach. I have shared this diagram with every teacher I've worked with as a representation of my strong belief in this premise:



A broad lens on learning enables teachers to build intelligently on students' existing foundations and erect new foundations for future study. A rich repertoire of knowledge empowers teachers to welcome rather than fear incisive questions and inspire students with fascinating mathematics rather than bore them with meaningless rituals.

**Premise 3: We must have a profound understanding of the mathematics we teach.** The traditional U.S. curriculum has been characterized as "a mile wide and an inch deep," because it traditionally attempted to "cover" so many topics while never "uncovering" any one topic in great depth. Consequently, an in-depth knowledge of the mathematics taught is often lacking. *Planting Seeds* will help foster a profound understanding of mathematics that will empower teachers to respond with fluency, fluidity, and flexibility to any classroom situation that arises.

**Premise 4: If we are comfortable, then we are no longer growing.** If teachers are comfortable with what they are teaching and how they have been teaching it, they become set in their ways and leave little room for growth. Leaving the comfort zone now and then is positive and productive. Discomfort leads to change, change begets growth, and growth engenders new learning and excitement. As you delve deeply into this book's explorations, you may feel a bit uncomfortable at times. The mathematics may require some effort to understand. But think of this experience as your own professional development, your adventure into new and stimulating mathematical



territory, where one day you will be able to lead your students with confidence, mindfulness, and knowledge.

**Premise 5: Raise the performance bar for students, and they will rise to meet it.** The National Mathematics Advisory Panel asserts, “American students have not been succeeding in the mathematical part of their education at anything like a level expected of an international leader” (2008, p. 3). The National Council of Teacher Quality (Greenberg & Walsh, 2008) laments that students in many foreign countries are two years ahead of U.S. students in mathematics. Therefore, I strongly believe that *all* students should have the right to challenging, high-level mathematics. If you find yourself thinking that the level of *Planting Seeds* is too advanced for your students, rather than worrying about what your students *won't be able* to understand, honor them with your belief that they *will be able* to think deeply about mathematics and engage in the problems. When students sense your confidence in them, they develop self-confidence and rise to meet your expectations.

If I had to explain algebra to a student, I would say: “Think of all that you know about mathematics. Algebra is about making it richer, more connected, more general, and more explicit. . . .”

Ricardo Nemirovsky (as quoted by Erick Smith, 2003)

## My Hope

For too many students, mathematics begins—in the world of arithmetic—as a meaningless set of numerical procedures, and then becomes—in the world of algebra—a meaningless set of symbolic procedures. It is my sincere hope that *Planting Seeds* will offer my readers more breadth and depth to the mathematics they already know as well as new insight into algebra’s many faces:

- Algebra as generalizing arithmetic
- Algebra as problem solving
- Algebra as examining structure
- Algebra as modeling real-world situations
- Algebra as studying relationships between quantities that change

Teachers should no longer think of algebra as separate from the mathematics they teach. This separation has been typical of traditional curricula that have long deprived students of powerful thinking about mathematics in the early grades and impeded them from learning algebra with understanding in the later grades. After reading this book, I trust my readers will see algebra not just as a garden of ideas but a garden in full bloom awash with vibrant colors.

## Notes

1. Reports by the International Commission on Mathematical Instruction (Stacey, Chick, & Kendal, 2004), the National Mathematics Advisory Panel (2008), the National Science Foundation (2007), and the RAND Mathematics Study Panel (2003).
2. <http://www.learner.org/resources/series98.html> (Scroll down to Workshops 7 and 8.)
3. <http://store.discoveryeducation.com/product/show/50647>
4. [http://www.media4math.com/algebra\\_nspirations.asp](http://www.media4math.com/algebra_nspirations.asp)
5. <http://www.univers-ti-nspire.com/forum/6-questions-sur-ti-nspire/1664-videos-ti-nspire-cas#1664> (Scroll down to view four videos.)

Mathematics is both an *object* of understanding and a *means* of understanding.

Thomas Romberg & James Kaput (1999)