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Nonlinear Methods for the Social Sciences

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Theories concerning attractors, bifurcations, chaos, fractals, and self-organization need to be tested eventually. Although the literature on nonlinear methods is vast, most of it has been written for applications that do not share the concerns or intellectual traditions of the social sciences, especially when short time series are concerned. This chapter is thus written for researchers who have a basic understanding of dynamical concepts and who want to test hypotheses concerning them in social psychological research, particularly for problems in group dynamics.

Fortunately, knowledge has evolved to a point where some important connections between fractal dimensions, Lyapunov exponents, chaos and other dynamics, Shannon information and entropy, catastrophes, and self-organization are now understood. These relationships can be exploited to produce a concise set of analytic tools that can be used with available software and with sufficient flexibility. Sprott (2003), Heath (2000), and Puu (2000), nonetheless serve as valuable supplementary resources on dynamics and time series analysis.

The following section of this chapter describes commonly used graphic techniques for nonlinear analysis. Next, I present some important relationships among dynamical constructs that give rise to useful statistical analysis. The statistical theory encompasses hypothesis construction, measurement theory, and two series of structural models that have wide flexibility; they involve continuously valued variables. The last section of this chapter describes analyses for nominally coded system states that are also changing in a time series.

Because the concentration of this chapter is on the analysis of real data, simulation techniques are not included here. Conceptual issues and relevant techniques for nonlinear analysis can be found in Elliott and Kiel (in press), Epstein and Axtell (1996), and Wolfram (2002). Chapter 16 of this volume (Nielsen, Sundstrom, & Halfhill, 2005) is devoted to simulation techniques for group research.

Graphic Techniques

Phase Portraits

The graphic of the control points' paths in the neighborhood of one or more attractors is called its *phase portrait.* Phase portraits can be drawn by plotting a behavior value at time *t* on the Y-axis against the value of the same behavior at time $t - 1$ on the X-axis. For more complex dynamics, the *change* in behavior from time $t - 1$ to *t* can be plotted on the Y-axis against the value of behavior differences at a previous pair of time frames ($t - 2$, $t - 1$) on the X-axis. A phase portrait of a fixed-point attractor would show trajectories moving into the center. A limit cycle would be round or elliptic.

Phase portraits need not be restricted to the one-variable case. One might then plot Y_t versus X_t, or ΔY versus ΔX. Often, however, researchers have a time series that should be projected into more than two dimensions, but exactly how many of these *embedding dimensions* are appropriate is unknown. Some work in progress uses methods based on principal components analysis for finding the most appropriate embedding dimension (Abarbanel, 1996; Abraham, 1997; Guastello & Bock, 2001). Until then, however, a convenient theorem to rely on states that all information about the dimensional complexity of a time series is contained in the time series itself. *Information* would include the embedding dimension and lag structures (Packard, Crutchfield, Farmer, & Shaw, 1980).

In the early days of applied chaos theory, some attempts were made to interpret dynamics of a real system (as opposed to a mathematically defined system) directly from an inspection of the phase portraits themselves (Kiel, 1994; Priesmeyer, 1992). The technique was quietly abandoned when it became clear that many of the time series were far too short, as in the case of Priesmeyer's (1992) examples, to determine whether chaos or anything else was taking place. In the case of sufficiently longer time series (e.g., those in Kiel, 1994), some examples showed visual differences that could be traced to real events, but most did not. The pretty attractors such as the one shown in Figure 14.1 were seldom obtained; the phase portraits tended to look like junk. Again, noise and embedding dimension were the top two reasons for this repeated outcome. Currently, the thinking is that analytic techniques should be applied first, and phase portraits should be drawn and compared afterward, if desired. This is similar to the thinking in conventional statistical analysis, where graphs do not substitute for statistical analysis; they just amplify the findings.

PoincarJ Sections

A PoincarJ section is a transverse slice of an attractor. It allows the viewer to inspect the interior of an attractor that is projected in three dimensions. A PoincarJ section of the Henon-Heiles attractor appears in Figure 14.1. Once again, the interpretive meaning of a PoincarJ section has not evolved for the same reasons associated with phase portraits.

Recurrence Plots

A recurrence plot is a graphic that shows the amount of patterning in a numeric time series for one variable. An example appears in Figure 14.2. The matrix of points is square, with *t,* the number of observations over time, occupying the two axes of the graph. The variate *X* is shown on the diagonal. Then, for each possible value of *X,* a point is plotted showing first and second time that particular value of *X* appears. If the same value appears again, another point is plotted showing the second and third time a particular value of *X* appears, and so on.

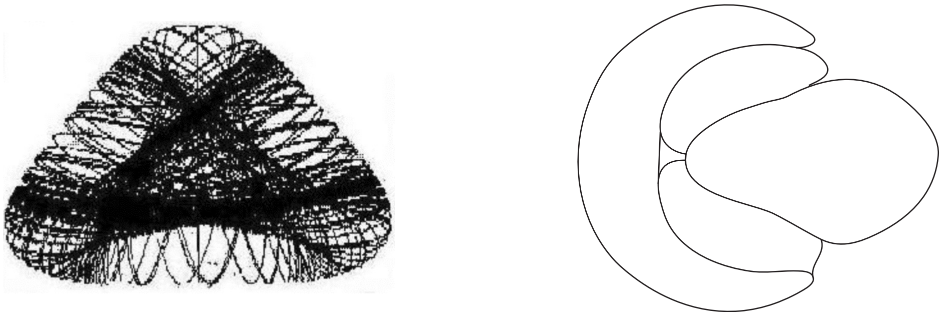

Figure 14.1 Henon-Heiles Attractor: (Left) Phase Portrait and (Right) Corresponding PoincarJ Section

The amount of patterning in a finished recurrence plot reflects the *correlation dimension* inherent in the time series. A correlation dimension is the degree of patterning in the data and is not directly related to the Pearson product-moment correlation or related statistics. The calculation and use of the correlation dimension are discussed in the next section of this chapter. For present purposes, we continue by saying that the "same" value of X is actually an arbitrarily small difference in value. Visual differences in plots can be obtained by simply increasing or decreasing the arbitrary definition of sameness.

Researchers who find value in recurrence plots or correlation dimensions in other ways compare the results of their analysis of a real time series with the results obtained from surrogate data. Surrogate data are usually produced by randomly shuffling the observations. This procedure will disrupt patterns that are detectable as a correlation dimension but preserve the autocorrelation among observations (Heath, 2000). A shuffled data set will produce a gray mass, as shown in Figure 14.2 in the case of extreme randomness, and a gray mass that is dense with diagonals in the case of autocorrelation, plus a great deal of noise.

The test of a hypothesis is again visual, but there are at least two targets to compare. Better grounding for any conclusions would require a more deliberate analysis of dynamical properties and a statistical analysis thereof.

Dimensions and Dynamical Footprints

It is now known that the basins of chaotic attractors are fractal in shape (Farmer, Ott, & Yorke, 1983; Mandelbrot, 1977, 1983). The dimensionality of an attractor is now regarded as a measure of the attractor's complexity and the complexity of the process that generated it. The exact relationship between dimensionality and the number of real variables that affect a process is not exact.

Fractal Dimensions

The calculation of fractal dimension is thus important for assessing the complexity of an attractor or other phenomena that are expressed in a spatial series or a time series. The concept of fractal dimension dates back to Hausdorff (1919) and was later modified by Mandelbrot (1983):

$$D_f = \lim_{1 \to 0} \frac{log\ [1/M(1)]}{\log 1}. \tag{1}$$

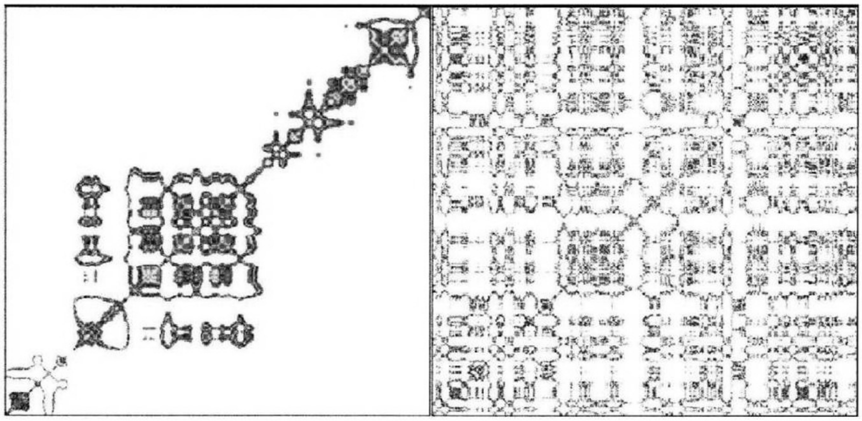

Figure 14.2 Recursion Plot for Heartbeat Data: (Left) Actual Data, (Right) Shuffled Data

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Imagine that a fractal image is covered with cubes with sides of length 1. M(1) is a function of the *embedding dimension,* ε. A true line will require ε = 1, a surface ε = 2, and volume ε = 3 and so forth. The number of squares required to cover a point is proportional to M(1), where

$$M(1) = 1/1^{-\varepsilon}. \qquad (2)$$

The *correlation dimension* has led to an often used algorithm developed by Grassberger and Procaccia (1983) for calculating attractor dimension. They defined dimension as:

$$D_c = \lim_{1 \to 0} \frac{log \, \|u_i\|_y}{\log 1} \qquad (3)$$

where 1 represents the diameter of a circle rather than the side of a square, and $\|u_i\|_y$ denotes the average value of 1/M(1) over all points in a time or spatial series of points. Although D_c was meant to approximate D_f, it is now known that $D_c < D_f$ (Girault, 1991; Kugiumtzis, Lillekjendlie, & Christophersen, 1994).

When applied to a time series, however, Equation 3 produces unreliable results depending on the amount of noise present in the data and whether the data are oversampled or undersampled (Theiler & Eubank, 1993). Because smooth mapping exists in the near neighborhood of an attractor (Wiggins, 1988), overly short time intervals would lead to a bias toward the conclusion that favored a linear interpretation of the data. Choice of time interval can seriously affect the definitions of attractors that one might extract from an analysis (Yee, Sweby, & Griffiths, 1991). When studying real systems (as we are in this chapter), Theiler and Eubank (1993) recommended that the rate of sampling be set relative to the physical properties of the system that is generating the data. For instance, if the system produces values every 2 seconds, a 2-second interval is appropriate. Similarly, if a particular type of economic data is generated every quarter of the year, then four observations per year would be appropriate.

Inverse Power Law

Another way to reach the correlation dimension is to rely on the principle of scale: If we take a frequency distribution of events and order the events by size, we would notice that the large examples of the event are relatively infrequent. The small examples would be more frequent. Next, let the size of the event be *X* in Equation 4, which is the *inverse power law:*

$$\text{Freq}(X) = a\,X^{-b} \qquad (4)$$

The parameters *a* and *b* can be estimated by nonlinear regression. The value of *b* that we obtain from that procedure is the fractal dimension. The examples of the inverse power law are widespread and varied according to Bak (1996) and West and Deering (1995).

The inverse power law function can also be obtained by transforming Equation 4 into Equation 5:

$$\log(\text{Freq}\,[X]) = -\,b\log[X] + a \qquad (5)$$

Although the two forms are mathematically equivalent, they are only statistically equivalent in the asymptotic case where there is no noise.

Lyapunov Exponents and Dimensions

Although chaotic attractors may exhibit fractional dimensionality, the presence of a fractional D_f is not a sufficient test of chaos. The property of sensitivity to initial conditions is still missing. The *Lyapunov exponent* is based on a concept of entropy; the exponent reflects the rate at which information that allows a forecast of a variable *y* is lost. It is calculated (Kaplan & Glass, 1995, p. 334–335; Puu, 2000, p. 157) by taking pairs of initial conditions y_1 and y_2 and their iterates one step ahead in time, which would be y_2 and y_3. If the ratio of absolute values of differences

$$L\,.\,|y_3 - y_2| \,/\, |y_2 - y_1| \qquad (6)$$

is less than 1.0, then the series is contracting. If the value of the function is greater than 1.0, then the function is expanding and sensitive dependence is present. The Lyapunov exponent, λ, is thus

$$\lambda = \ln\,[L] \qquad (7)$$

For an ensemble of trajectories in a dynamical field, Lyapunov exponents, λ_i, are computed for all values of *y*. If the largest value of λ is positive, and the sum of λ_i is negative, then the series is chaotic.

The calculation of Equation 7 is made on the entire time series and averaged by taking the geometric mean of *N* values where *N* is the last entry in the time series. It is also possible to rearrange the terms:

$$\lambda = (1/\text{N}) \sum_{N=1}^{N} \ln\,(\text{L})\,| \qquad (8)$$

The foregoing calculations generalize as:

$$y = e^{\lambda t}, \qquad (9)$$

which is actually insensitive to initial conditions. A positive value of λ indicates an expanding function, which is to say, chaos. A negative λ indicates a contracting process, which could be a fixed point or limit cycle attractor. Dimension, D_L becomes a function of the largest value of λ in the series (Frederickson, Kaplan, Yorke, & Yorke, 1983; Kugiumtzis et al., 1994; Wiggins, 1988):

$$D_L = e^{\lambda} \qquad (10)$$

The statistical approach for determining 1 is presented subsequently in this chapter. The statistical approach is a variant of Equation 9, and restores sensitivity to initial conditions.

The Lyapunov exponent is an indicator of turbulence (Ruelle, 1991), such as the turbulence that occurs in air and fluid flows. The relationships among entropy and information as defined by Shannon (1948), topological entropy, and the Lyapunov exponent are considered later in this chapter in the context of nominally coded system states.

Self-Organizing Dynamics

The mathematical elements of self-organizing dynamics fall into a few different categories depending on whether one is working from the vantage point of sandpile dynamics (Bak, 1996), rugged landscapes (Kauffman, 1993, 1995), synergetics (Haken, 1984, 2002), agent-based models (Epstein & Axtell, 1996; Holland, 1995), or cellular automata (Wolfram, 2002). All forms of self-organization depict the origins of order in a system as arising from the local interaction of system elements. All local interactions, furthermore, can be characterized by the flow of information between system elements.

Sandpiles

The sandpile dynamic (Bak, 1996) characterizes a system as one that changes from a small and formless entity to more complex distribution of entities as interacting elements are added. At a critical point in time, the single sandpile incurs an avalanche that results in a $1/f^b$ distribution of large and small piles. The analysis of a $1/f^b$ distribution and the relationship between the fractal dimension and chaos were elaborated above. It is noteworthy to add at this point that a $1/f^b$ distribution is, in essence, an exponential distribution. Exponential distributions are fundamental to several forms of nonlinear dynamical statistical analysis, as elaborated below.

Rugged Landscapes

The rugged landscape model of self-organization (Kauffman, 1993, 1995) originated with the spin-glass simulation. In essence, if we take a sample of molecules that are heterogeneous for their electron configurations, put them in a suitable container, and introduce energy to spin the container around, we eventually obtain a separation of molecular variations such that similar forms cluster together. In the evolutionary counterpart to this metaphor, a heterogeneous species or organism on a mountaintop encounters an environmental force that requires the colony of organisms to separate and find new ecological niches on the new landscape. The erstwhile heterogeneous species now clusters into smaller and more homogeneous groups.

The distribution of new homogeneous groups is characterized by the *NK* distribution, where a large number of new groups (N) share a small number of common characteristics (K), whereas a smaller number of new groups share larger numbers of common characteristics. The *NK* distribution is a member of the exponential family, but unlike a simple exponential distribution, it is actually multimodal (Guastello, 1998a). Fortunately, catastrophe theory (Guastello, 1995; Thom, 1975; Zeeman, 1977) has produced not only a series of interesting mathematical models that are applicable to self-organizing dynamics, but also a series of statistical models for multimodal distributions within the exponential family. Catastrophe models might express the differentiation of a heterogeneous system state into multiple homogeneous states. They can also model the discontinuous movement of an entity from one ecological niche to another.

Phase Shifts

The synergetic approach to self-organization describes phase shifts and coupled dynamics (Haken, 1984, 2002). A phase shift, such as the transition of a liquid to a gas, generalizes as a qualitative change in the nature of the interaction among molecules or interaction among people in a social system (Galam, 1996). The relationship between phase shifts and catastrophes was known long ago (Gilmore, 1981). The master equation for a phase shift is generally written as:

$$f(y) = y^4/4 - ky^2/2 - ay, \qquad (11)$$

where *y* is the behavior and *a* is a control parameter that governs how close a person or object is to the critical point of behavioral transition. In many applications to self-organizing systems, *k* is a constant that depicts the size or suddenness of the behavioral transition. If *k* were a variable denoting that some people make bigger changes than others, then we obtain a full-fledged cusp catastrophe model instead of just a single slice of it. Note that no one really cares about the constants 4 and 2 in Equation 11 for reasons that should be clear before the conclusion of this chapter.

A cusp catastrophe model is shown in Figure 14.3, where *a* is the asymmetry parameter and *k* is the bifurcation parameter (which is, henceforward denoted as the variable *b*). The model for the changes of behavior in the system forms the response surface that is shown in Figure 14.3. The response surface depicts two stable states of behavior that are separated by a bifurcation manifold that has critical points of instability lined up on its outer edges. The equation of the response surface is the derivative of Equation 11:

$$df(y)/dy = y^3 - by - a. \qquad (12)$$

However, the implicit derivative as a function of time is perhaps more intuitively appealing (Guastello, 1995):

$$dy/dt = y^3 - by - a. \qquad (13)$$

Coupled Dynamics

Coupled dynamics are situations where *y* is a nonlinear function of itself and *x*, and *x* also displays nonlinear dynamical properties. In Haken's (1984) terminology, *x* would be a *driver* variable, and *y* would be a *slave*. A driver is a dynamical subsystem that produces output that greatly affects the dynamics of another subsystem. The latter subsystem is the slave, which, although it contributes dynamics of its own, produces dynamics that are greatly influenced by the driver. Driver-slave dynamics form the basis of the hierarchical dynamical system. I will return to coupled dynamics after simpler dynamical systems are explained.

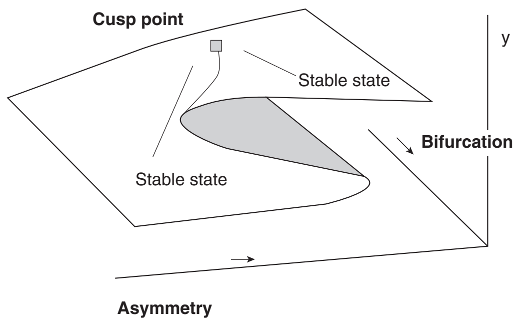

Figure 14.3 Cusp Catastrophe Model

Edge of Chaos

The basic concept (Waldrop, 1992) is that systems poised on the edge of chaos could self-organize at any moment in response to a critical stimulus of some sort. A critical stimulus would be one that could not be assimilated by the existing structure of the system, and thus a new structure would be necessary for an effective response. The effective response may involve unraveling the existing subsystems. Repeated unraveling and reorganization of a system would be expected occasionally from a functional complex adaptive system.

It is noteworthy that the earliest studies of chaotic behavior in physiological systems relied on computations of the fractal dimension, which is a relative of the Lyapunov exponent; a comparison of the relative assets and limitations of the two indicators is beyond the scope of this chapter. Some qualitative trends in those studies are relevant nonetheless: Studies of cell tissue, electroencephalograms (EEGs), and electrocardiograms (EKGs) indicate that greater irregularity (turbulence, complexity) appears in the output (or cell morphology) from healthy systems. Unhealthy systems gravitate toward periodic and simplistic output (Goldberger, Peng, Mietus, Havlin, & Stanley, 1996; Hornero, Alonso, Jimeno, Jimeno, & Lopez, 1999; Meyer et al., 1998; Sabelli & Kauffman, 1999). This trend has been extended to organizational behavior (Dooley, 1997) and communication dynamics within families (Pincus, 2001).

For analytic purposes, the point here is that a system that is in the process of self-organizing can exhibit a chaotic dynamic and a self-organizing dynamic at the same time. The case of a dual function in a group learning situation will be considered later in this chapter. Therein, we see the dynamics of both chaos and learning.

The Structural Equations Technique

The structural equations technique begins by defining a model in the form of an equation, then testing it statistically with real (as opposed to simulated) data. The analysis separates the deterministic portion of the data from noise. *Noise* here denotes that portion of the data variance that is not explained by the deterministic equation. Social scientists will recognize this model-versus-noise approach as "business as usual." This technique contrasts with a prevailing habit in the physical sciences, which works in the opposite fashion: Separate the noise first, then make calculations on what remains (e.g., Kanz & Schreiber, 1997). Although social scientists have adopted a filter-first approach to data analysis (e.g., Heath, 2000), filtering is clearly not recommended for the techniques described below.

Two series of hierarchical models are considered here. The first is the catastrophe models for discontinuous change (Guastello, 1992a, 1995, 2002). The second set involves exponential models for continuous change and includes a test for the Lyapunov exponent, which distinguishes between chaos and nonchaotic dynamics. The latter set was introduced by Guastello (1995) and built on previous work by May and Oster (1976), Wiggins (1988), and numerous other contributors to the field of nonlinear dynamical systems.

Each model in a hierarchy subsumes properties of the simpler models. Each progressively complex model adds a new dynamical feature. This chapter covers models involving only one order parameter (dependent measure). Two-parameter models can be tested as well, but the reader is directed to Guastello (1995) to see how those extrapolations can be accomplished.

The following sections of this chapter address the type of data and amounts that are required; probability functions, location, and scale; the structure of behavioral measurements; the catastrophe model group, which can be tested through power polynomial regression; the exponential series of models, which can be tested through nonlinear regression; and catastrophe models that are testable as static probability functions through nonlinear regression.

Types and Amount of Data

The procedures that follow require dependent measures (order parameters) that are taken at a minimum of two points in time. One may have many entities that are measured at two points in time, or one long time series of observations from one entity. Alternatively, one may have an ensemble of shorter time series taken from several entities.

In general, it is better to have a smaller number of observations that cover the full dynamical character of a phenomenon than to have a large number of observations that cover the underlying topology poorly. Because these are statistical procedures, all the usual rules and caveats pertaining to statistical power apply. The simplest models can be tested with 50 data points and sometimes fewer, if there are (a) good models of the phenomenon in question, (b) reliable measurements, and (c) only one or two regression parameters to estimate. In all cases, more data is better than less data so long as the data are actually covering all the nonlinear dynamics that are thought to exist in the system.

Statistical Power

The calculation of statistical power for ordinary multiple regression depends on the intended effect size, overall sample R^2, population R^2, the number of independent variables, the degree of correlation among the independent variables, and occasionally the assumption that all independent variables are equally weighted. Not surprisingly there are different rubrics for determining proper sample size.

According to Cohen (1988), to detect a medium effect size of .15 for one of the independent variables with a power of .80, the appropriate sample size is 52 plus the number of estimated parameters (Maxwell, 2000). Thus, the required sample size would be 58 observations for a six-parameter model. According to a similar rubric by Green (1991), a sample size of 110 should detect an effect size of .075 for one independent variable with a power of .80. The current sample sizes would thus detect effect sizes of .07 and .04, respectively, with a power of .80. However, neither rubric takes into account that the odds of finding a smaller partial correlation increase to the extent that the overall R^2 is large. According to Maxwell (2000), the odds of detecting one of the effects within a multiple regression model drops sharply as the correlations among independent variables increase.

One should bear in mind that the calculation of statistical power for nonlinear regression is still generally uncharted territory. It is thus necessary to rely on the rubrics for linear models. If there is sufficient statistical power for the linear comparison models, which in the past have been generally weaker in overall effect size than nonlinear models when the nonlinear model was held true, there should not be much concern with the statistical power of the nonlinear models. On the other hand, the power for specific effects within a nonlinear model probably depends on whether the regression parameter is associated with an additive, exponential, multiplicative, or other type of mathematical operator.

Optimal Time Lag

Put simply, the time lag between observations is optimal if it reflects the real time frame in which data points are generated. For instance, catastrophe models are usually lagged "before" and "after" a discrete event. Macroeconomic variables such as inflation and unemployment rates are studied best at lags equal to an economic quarter of the year (e.g. Guastello, 1999b). Economic policies are usually implemented on a quarterly basis, even if some of the important indicators are posted monthly.

Probability Density Functions

It is convenient that any differential function can be transformed into a probability density function (pdf) using the Wright-Ito transformation. The variable y in Equation 14. is a dependent measure that exhibits the dynamical character under study; y is then transformed into z with respect to *location* (λ) and *scale* (σ_s, Equation 15).

$$\mathrm{pdf}(z) = \xi \exp[\,-\mathrm{l}\, f(z)]; \qquad (14)$$

$$z = (y - \lambda) / \sigma_s . \qquad (15)$$

Location. In most discussions of probability functions, *location* refers to the mean of the function. In dynamics, the pdf is a member of an exponential family of distributions and is asymmetrical, unlike the so-called normal distribution. Thus, the location parameter for Equation 14 is the lower limit of the distribution, which is the lowest observed value in the series. The transformation in Equation 15 has the added advantage of fixing a zero point and thus transforming measurements with interval scales (common in the social sciences) to ratio scales. A fixed location point defines where the nonlinear function is going to start.

Scale. The scale parameter in common discussions of pdfs is the standard deviation of the distribution. The standard deviation is also used here. The use of the scale parameter later on while testing structural equations serves the purpose of eliminating bias between two or more variables that are multiplied together. Although the results of linear regression are not affected by values of location and scale, nonlinear models are clearly affected by the transformation.

Occasionally, one may obtain a better fit using the alternative definition of scale in Equation 16, which measures variability around statistical modes rather than around a mean:

$$\sigma_s = [q \sum_{y=1} \sum_{m=1} (y_{m-}^{m+} - mi)^2] \,/\, N - M \qquad (16)$$

To use it, the distribution must be broken into sections, each section containing a mode or an antimode. The values of the variable around a mode will range from *m–* to *m+* as depicted in Equation 16.

Corrections for location and scale should be made on control variables as well as dependent measures. The ordinary standard deviation is a suitable measure of scale for control variables.

Structure of Behavioral Measurements

In the classic definition, a measurement consists of true scores (T) plus error (e). The variance structure for a population of scores is thus:

$$\sigma^2(X) = \sigma^2(T) + \sigma^2(e). \qquad (17)$$

The classical assumption is that all errors are independent of true scores and all other errors.

In nonlinear dynamics, our true score is the result of a linear (L) and nonlinear deterministic process (NL), dependent error (DE), and independent error (IIDE):

$$\sigma^2(X) = \sigma^2(L) + \sigma^2(NL - L) + \sigma^2(DE) + \sigma^2(IIDE) \qquad (18)$$

"Independent error" in conventional psychometrics is known as independently and identically distributed (IID) error in the nonlinear dynamical systems literature. Dependent error is commonly observed in time series applications where the amount of error is dependent on the value of *X*. In dynamical systems, a modicum of error that occurs at one point in time becomes part of *X* on the next iteration and is thus amplified or computed by the system along with the true score component of X. It is significant that the non-IID (DE) error is a result of the nonlinear deterministic process (Brock, Hseih, & LeBaron, 1991).

Catastrophe Models

The analysis that follows requires the polynomial form of multiple linear regression. The analysis can be performed with most any standard statistical software package. Several concepts for hypothesis testing carry through to subsequent analyses of other dynamics.

The set of catastrophe models was the result of the classification theorem by Thom (1975): Given certain constraints, all discontinuous changes in events can be described by one of seven elementary models. Four of the models contain one order parameter; this is the cuspoid series: fold model (one control parameter), cusp model (two control parameters), swallowtail model (three control parameters), and butterfly model (four control parameters). The remaining three models, known as the umbilic group, contain two order parameters. The descriptions that follow pertain to the cusp but generalize readily to the cuspoid group.

The process of hypothesis testing begins with choosing a model that appears to be closest to

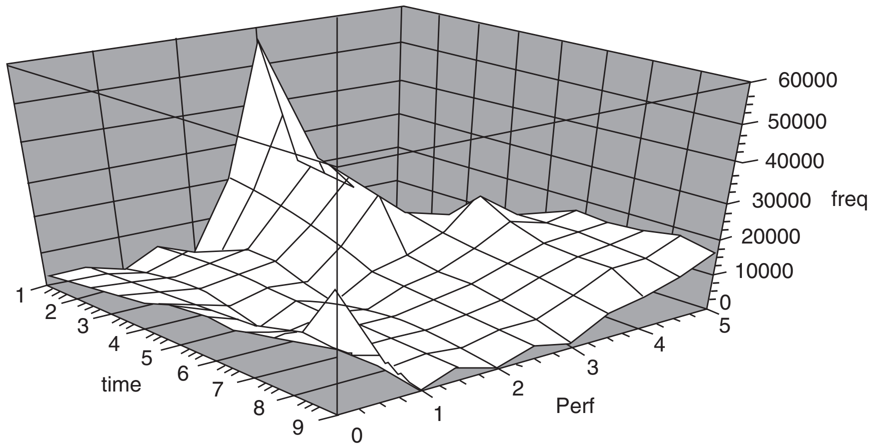

Figure 14.4 Cusp pdf From a Multistage Personnel Selection Application

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the phenomenon under investigation. Because the cusp is the most often used model, the following remarks are framed in terms of the cusp model, which is described in Figure 14.3. Note that two or more experimental variables may be hypothesized for each control parameter without changing the basic model or analytic procedure. Examples of analyses using the polynomial regression method for catastrophes date back to Guastello (1982). Recent examples, however, can be found in Guastello (1995, 2002), Guastello, Gershon, and Murphy (1999), Clair, (1998); Lange (1999), and Byrne, Mazanov, and Gregson (2001).

Nonlinear statistical model. The deterministic equation for the cusp is shown in Equation 19 and followed by its pdf using the Wright-Ito transformation in Equation 20:

$$\delta f(y)/\delta y = y^3 - by - a, \qquad (19)$$

$$\text{pdf}(z) = \xi\, e^{[-z_4/4 + bz_2/2 + az]} \qquad (20)$$

Figure 14.4 depicts a cusp pdf that was produced from real data (Guastello, 2002, p. 136).

Next, we take the deterministic equation for the cusp response surface and insert regression weights and a quadratic term:

$$\Delta z = \beta_0 + \beta_1 z_1^{\,3} + \beta_2 z_1^{\,2} + \beta_3 b z_1 + \beta_4 a, \qquad (21)$$

The quadratic term is an additional correction for location. The dependent measure Δz denotes a change in behavior over two subsequent points in time.

Several hypotheses are being tested in the power polynomial equation (Equation 21). The F test is used for the model overall; the R^2 coefficient can be retained and saved for later use. There are t tests on the beta weights; they denote which parts of the model account for unique portions of variance.

Some model elements are more important than other elements. The cubic term expresses whether the model is consistent with cusp structure; the correct level of complexity for a catastrophe model is captured by the leading power term. If there is a cusp structure, then one must identify a bifurcation variable as represented by the $\beta b z_1$ term. A cusp hypothesis is not complete without a bifurcation term; shabby results may be expected otherwise. The asymmetry term βa is important in the model, but failing to find one does not negate the cusp structure if the cubic and bifurcation elements are present. The lack of an asymmetry term only means that the model is not complete.

The quadratic term is the most expendable. It is not part of the formal deterministic cusp structure. Rather, it is an additional correction for location (Cobb, 1981a). In the event that unique weights are not obtained for all model components, the quadratic term can be deleted and the remaining elements tested again.

Note the procedural contrast with linear regression analysis: In common linear regression, when a variable does not attain a significant weight, we simply delete that variable. In nonlinear dynamical systems, we delete variables based on their relative importance to the structural model. In linear analyses, only a linear structure is under consideration, so particular variables are then kept or discarded. In nonlinear analyses, different variables may be playing different structural roles.

Linear comparison models. In Equations 22 and 23, we compare the R^2 coefficients against the R^2 that was obtained for the cusp:

$$y_2 = B_0 + B_1 y_1 + B_2 a + B_3 b, \tag{22}$$

$$\Delta y = B_0 + B_1 a + B_2 b. \tag{23}$$

Next, the elements of the cusp model are evaluated. If all the necessary parts of the cusp are significant, and the R^2 coefficients compare favorably, then a clear case of the cusp has been obtained.

Exponential Model Series

This section describes a series of models that exhibit continuous but nevertheless interesting change. The model structures are functions of the Naperian constant *e*. They produce, among other things, the Lyapunov exponent, which is a test for chaos and a value comparable to the fractal dimension.

Nonlinear regression is required to test this series of models. Nonlinear regression may be familiar to biologists, but it is probably much less familiar to social scientists at the present time. This schism in uptake of nonlinear regression is probably related to the dearth of explicit nonlinear models in the social sciences prior to the advent of nonlinear dynamical systems. The hierarchical series of models ranges from simple to complex as follows: (a) simple Lyapunov exponent, (b) Lyapunov with additional fitting constants, (c) May-Oster model with the bifurcation parameter unknown, (d) model with an explicitly hypothesized bifurcation model, and (e) models with two or more order parameters.

Examples of analyses using the nonlinear regression method for chaos and related exponential models date back to Guastello (1992b). More recent examples can be found in Guastello (1995, 1998b, 1999a, 1999b, 2001, 2002), Guastello and Philippe (1997), Guastello and Guastello (1998). Guastello and Johnson (1999), Guastello, Johnson, and Rieke (1999), Guastello and Bock (2001), Rosser, Rosser, Guastello, and Bond (2001), and Guastello and Bond (in press). For examples that compare dimensionality estimates made through nonlinear regression with values obtained by other means, see Johnson and Dooley (1996) and Guastello and Philippe (1997).

Lyapunov models. The simplest model predicts behavior z_2 from a function of z_1. Note that the corrections for location and scale apply here as well:

$$z_2 = e^{(\theta_1 z_1)} \tag{24}$$

The nonlinear regression weight θ_1 located in the exponent is also the Lyapunov exponent. It is a measure of turbulence in the time series. If θ_1 is positive, then chaos is occurring. If θ_1 is negative, then a fixed point or periodic dynamic is occurring. D_L is an approximation of the fractal dimension (Ott, Sauer, & Yorke, 1994):

$$D_L = e^{\theta_1}. \tag{25}$$

The second model in the series (Equation 26) is the same as the first except that two constants have been introduced to absorb unaccounted variance. The Lyapunov exponent is now designated as θ_2:

$$z_2 = \theta_1 e^{(\theta_2 z_1)} + \theta_3 . \tag{26}$$

In nonlinear regression, it is necessary to specify the placement of constants in a model. Unlike the case in the general linear model, constants in nonlinear models can appear anywhere at all. Hence θ_1 and θ_3 are introduced in Equation 26.

The suggested strategy here is to start with the second model (Equation 26). If statistical significance is not obtained for all three weights, delete θ_1 and try again. If that result is not good enough, drop the additive constant θ_3 and return to the simplest model of the series (Equation 24).

Bifurcation models. The third level of model is shown in Equation 27. Note the introduction of z_1 between θ_1 and *e:*

$$z_2 = \theta_1 \, z_1 \, e^{\theta_2 z_1} + \theta_3. \tag{27}$$

Equation 27 tests for the presence of a variable that is possibly changing the dynamics of a model. For instance, some learning curves may be sharper than others. A positive test for the model indicates that a variable is present, but its identity is not yet known.

The computation of dimension is similar to that of previous exponential models, except that a value of 1 must be added to account for the presence of a bifurcation variable:

$$D_L = e^{\theta_2} + 1. \tag{28}$$

At the fourth level of complexity, the researcher has a specific hypothesis for the bifurcation variable, which is designated as *c* in Equation 29:

$$z_2 = \theta_1 c \, z_1 \, e^{\theta_2 z_1} + \theta_3. \tag{29}$$

Linear contrasts. As in the case of the catastrophes, we test the R^2 for the nonlinear regression model against that of the linear alternatives, such as

$$y_2 = B_0 + B_1 y_1 \tag{30}$$

or

$$y_2 = B_0 + B_1 t, \tag{31}$$

where *t* is time.

Tips for using nonlinear regression. On the model parameter command line (or comparable command in statistical packages), the names of the regression weights are specified with initial values. Researchers either use the initial values of 0.5 or pick their own. When in doubt, initial weights can be given equal value. Often, it does not matter whether the iterative computational procedure starts off with equal weights or not. If the model results are not affected by the initial values, then the resulting model is more robust than otherwise would be the case.

In cases where there is an option to choose constrained versus unconstrained nonlinear regression, the unconstrained option is typically the default. Constraints indicate that the researcher expects the values of parameters to remain within numerical boundaries that have been predetermined. Occasionally, there may be a good rationale for containing parameters, but they are generally specific to the problem.

If there is an option to choose least squares or maximum likelihood error term specification, be forewarned: Maximum likelihood is more likely to capitalize on chance aspects of the pdf and is thus more likely to return a significant result. It may be better to use least squares for this reason.

If the results of a nonlinear regression analysis are so poor that they produce a negative R^2, researchers need not be alarmed. The negative R^2 can be treated as if it were .00.

When testing for significance, the tests on the weights are very important. Some researchers value them more than the overall R^2. Tests for weights are made using the principle of confidence intervals. An alpha level of $p < .05$ is regarded as unilaterally sufficient. A nonsignificant weight with a high overall R^2 could be the result of a high correlation among the parameter estimates; this condition is akin to multicollinearity in ordinary linear models.

Testing Catastrophes Through Nonlinear Regression

The two strategies previously delineated can now be combined for some special circumstances. Sometimes, one might obtain a pdf that bears a strong resemblance to that of an elementary catastrophe, and it is logical to frame a hypothesis as to whether that association is true or false. In another situation, a catastrophe process may be occurring, but all the time – 1 measurements are the same value of 0.00. In both types of situations, it would be good to test a hypothesis concerning the catastrophe distribution. Examples of analyses using the nonlinear regression method for testing catastrophe pdfs are sparse, although the method was proposed by Cobb some time ago (1981a, 1981b). More recent examples can be found in Hanges, Braverman, and Rentch (1991), Guastello (1998a, 2002), and Zaror and Guastello (2000).

Dual Functions

Three interesting cases are known where two dynamical functions were operating concurrently in a system. One such example was identified in a creative problem-solving study that involved a periodic driver and a chaotic slave (Guastello, 1998b). The resulting model took the form:

$$z_2 = \mathrm{f}(z_1) + \beta_1 \mathrm{a} + \beta_2 b + \beta_3 c, \tag{32}$$

where g(z) was an exponential function from the model series above; a, b, and c were control parameters; and β_i were regression weights that held the model together. The primary function f(z) displayed a positive Lyapunov exponent. Control variables a and b were categorical variables that contributed additive explanatory variance. Variable c, however, was itself a different exponential function over time with a negative exponent.

The interpretation of driver and slave had to be made in the context of the problem. The important result from the study, however, was that two dynamics were involved and that it was possible to state the effects produced by each variable. A procedural annoyance, however, was that the linear comparison model contained four additive elements, of which one was still nonlinear. The R^2 for the linear comparison model was relatively high, but not greater than the R^2 for the nonlinear model. Thus, the linear model "cheated" a bit.

A second interesting example was identified in a group coordination learning experiment (Guastello & Guastello, 1998). The group readily learned the coordination task for two of the experimental conditions. Exponential models with the bifurcation effect characterized their behavior over time. Figure 14.5 depicts the relationship between learning curves and a cusp bifurcation manifold. The exponential model characterizes only a slice of the full surface that pertains to the behavior of the groups in the study.

One experimental condition in the coordination study was very difficult for the participants. The first function to be extracted was a chaotic function, which indicated in this context that the adaptive self-organizing response had not kicked in for some of the groups. When that variance was removed from the behavioral

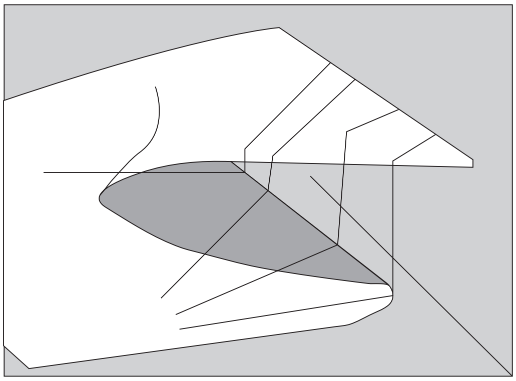

Figure 14.5 Learning Curves Represented as Slices From a Cusp Catastrophe Manifold

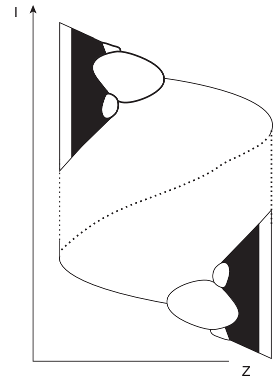

Figure 14.6 Compound Dynamic for Hysteresis Between Two Chaotic Attractors

variable, however, the analysis of the residual showed a second function that contained a negative exponent. Thus, an incomplete self-organized response may contain two kinds of functions, and an analysis of residuals would be required to find them both.

The third interesting case involved a combination of a cusp catastrophe model and chaotic attractors (Rosser et al., 2001). The stable states, or attractors, in a catastrophe model are fixed point attractors, although the presence of limit cycles would not usually impair the analysis. The theory that Rosser et al. were working with indicated a cusp manifold for hysteresis, but the attractors were chaotic. A graphic representation of this combination of dynamics appears in Figure 14.6.

They did not know what the control variables could be, so they simply analyzed the dynamics of their two economic indicators (annual infrastructure investment by the Soviet government and the annual investment in construction by the Soviet government) using the exponential series. One variable displayed a bifurcation effect and a negative exponent. The other displayed no bifurcation and a positive exponent. Although the dynamics that were obtained did not show a clear illustration of what was hypothesized in simpler terms, the dynamics did display portions of the intended dynamics. Ideally, they would have found a chaotic function, a bifurcation effect, and a dimensionality in the neighborhood of 3.0.

Perhaps, longer data sets with control variables would have produced different results; one variable consisted of 53 annual numbers, and the other consisted of 27 annual numbers. The epoch of Soviet history that they were studying ended, however, and they were compelled to extract meaning out of what data actually existed.

Symbolic Dynamics and the Method of Orbital Decomposition

Symbolic dynamics is an area of mathematics that finds patterns in series of qualitative data. Furthermore, the elementary patterns can be treated like qualitative states themselves and subjected to pattern detection for higher order patterns. This is the basic concept behind Turing's Universal Computational Machine. The approach is ideally suited to analyzing chaotic and related complex nonlinear dynamics (Robinson, 1999), particularly when self-organizing phenomena are likely to emerge (Crutchfield, 1994). The process is usually applied to continuously valued time series. Events such as spikes and small or large up-trends and down-trends are coded nominally (e.g., with letter codes A, B, C, D, etc.) and then analyzed. Several computational procedures have been advanced; some include computations of Shannon's entropy or other dynamical indicators.

Numerous biomedical applications of symbolic dynamics techniques have been reported recently (Yamada et al., 2000; Yeragani, Nadella, Hinze, Yeragani, & Jampala, 2000). In principle, symbolic dynamics might be applied to any situation where nonlinear behavior or living systems are involved (Arecchi, 2001). Within psychology, symbolic dynamics have been recommended for the study of firing patterns of individual neurons within a network (Lewis & Glass, 1992), activation of neural circuits involved in memory processes (Guastello, Nielson, & Ross, 2002), linguistics problems (Sulis, 1998), and the analysis of conversations between a psychotherapist and a client (Rapp & Korslund, 2000). Actual applications have been published on topics concerning artificial grammars (Bollt & Jones, 2000), creative problem-solving groups (Guastello, 2000; Guastello, Hyde, & Odak, 1998), and family systems dynamics (Pincus, 2001).

The foregoing techniques vary in their method for determining symbol sequences and the length of those sequences. They also vary in the extent to which they produce results that interface with other dynamical concepts such as topological entropy and the Lyapunov exponent. The latter three studies involve the method of orbital decomposition, which is assisted by a statistical analysis, unlike the other current offerings. The method of orbital decomposition is decribed next.

The method of orbital decomposition (Guastello et al., 1998) is based on the principle that a minimum of three coupled oscillators are required to produce chaos (Newhouse, Ruelle, & Takens, 1978). The procedure does not require the presence of chaos; it merely accommodates systems of sufficient complexity to qualify as chaos. The procedure requires three calculations in parallel: Shannon entropy (*H*S), topological entropy (*H*T), and a likelihood χ^2 test for strings of responses of varying length *C*. The calculations provide measures of dimensional complexity, the determination of an optimum behavior string length, a set of behavior strings with associated probabilities, and a chi-square test that provides a measure of fitness for the string structures.

Shannon (1948; Ott et al., 1994) entropy (*H*S) is defined in Equation 33, where *p*i is the probability associated with one categorical outcome in a set of *r* categories

$$HS = \sum_{i=1}^{r} pi \ln (1/pi). \tag{33}$$

The second calculation, topological entropy (*H*T), is based on strings, or hypothetical orbits, of length *C*. *C* takes on a small range of integer values beginning with 1. For $C = 1$, a transition matrix, M^C is created, which is square, and $r \times r$ in size. Each cell entry is binary and indicates whether a particular behavior category is followed in time by any other behavior category. Its diagonal entries indicate whether an outcome is followed by itself in a consecutive period of time. Topological entropy is a function of the trace of M^C (Lathrop & Kostelich, 1989, p. 4030):

$$H_T = \lim_{c \to 4} (1/C) \log2\text{tr}(M^C) \tag{34}$$

As the string length goes to infinity, *H*T approaches the base – 2 logarithm of the trace of M^C, and is in turn equal to the maximum Lyapunov exponent, which is also the largest eigenvalue of M^C. Dimensionality is, therefore,

$$D_L = e^{HT} \tag{35}$$

The construction of M^C is repeated for all $C >$ 1, and Equations 33 to 35 are calculated for each. For $C = 2$, one axis of M^C represents all possible pairs of categories, although some possible combinations might not actually appear in the data if their combinatorial probability is too low.

The third calculation is the χ^2 for goodness of fit, and it is also carried out for each value of *C* used. The essential question posed by the test is whether the behavior strings observed in the data occur at rates different from chance, where chance is simply the combinatorial probability of each categorical element in the string. The likelihood χ^2 is preferred over the Pearson variety because the expected (*F*Ex) and observed (*F*Ob) frequencies for many elementary categories and strings are small:

$$\chi^2 = 2\,\Sigma\, F\text{Ob} \ln(F\text{Ob}/F\text{Ex}). \qquad (36)$$

Given *N* strings of length *C* in a set, the expected frequency of string X-Y-Z is

$$F\text{Ex} = PXPYPZN. \qquad (37)$$

Finally, ϕ^2 coefficients are calculated for all χ^2 tests to provide a measure of variance accounted for by the observed strings for all lengths *C*:

$$\phi^2 = \chi^2/N. \qquad (38)$$

Optimal *C* is determined as the length of a string one step before the step at which *HT* drops to 0.00. Having determined the optimal string length, it is possible to describe the contents and distribution of strings with that length; these were used as a basis of comparison with other conversations, along with the associated values of *C, HT*, and *HS*. The array of strings identified at this stage could be analyzed for hierarchical dynamics using a repetition of the process just outlined (cf. Crutchfield, 1994).

H_T is in indicator of the amount of information produced by the underlying (neuronal) process. H_T decreases as *C* increases; it is the primary indicator of the asymptotic limit of C. A comparison of the asymptotic H_T and *C* values across experimental conditions would indicate which neuronal processes are more complicated than others and how.

To date, the available applications of the orbital decomposition technique have pertained to the analysis of creative problem-solving conversation (Guastello, 2000; Guastello et al., 1998) and conflict resolution conversations among members of dysfunctional families (Pincus, 2001). Guastello et al. (1998) analyzed a string of more than 500 responses in one real time problem-solving session used in the study just described. Responses were coded in one of nine possible types of input: requesting information, giving information, tension reduction, clarifying responses and ideas, gate keeping, initiating, following, harmonizing, and unclassified responses. Guastello et al. found that sequences of four responses emerged and that the sequences could not be reduced to the simple effects of combinatorial probabilities. In a subsequent study (Guastello, 2000), the elementary conversational units were themselves combinations of conversation contribution types. Table 14.1 (from Guastello, 2000) illustrates the array of data that led to the conclusion that, for a particular coding scheme that was applied to a creative problem-solving conversation, the optimal string length consisted of sequences of 3. Thereupon, the trace of the pattern matrix was 4, and other relevant computations appear in the same row. The ϕ_2 value indicated that 95% of the variance in states within the original data set was accounted for by the strings produced at $C = 3$.

Once the correct length of *C* has been determined, it is then possible to interpret the strings themselves; each string, which corresponds to an orbit, contains three qualitative states in the example above. A frequency distribution of string frequencies will inevitably show that some strings occur very often whereas others occur less frequently. We speculate that the frequency distribution of string frequencies will correspond to an $1/f^a$ distribution, which is typically indicative of a fractal or self-organizing process (Bak, 1996; West & Deering, 1995). It is noteworthy that some strings that are possible do not actually appear. It is currently recommended

Table 14.1 Trace of Binary M^c, Topological Entropy, Dimensionality, χ^2, ϕ^2, and Shannon Entropy for Strings of Length 1 through 4; Analysis of a Group Problem-Solving Conversation

C	$Tr(M^c)$	H_T	D_L	χ^2	df	N	ϕ^2	H_S
1	5	2.322	10.196	46.92	8	81	.58	2.239
2	4	1.000	2.718	34.84	36	80	.44	3.478
3	4	0.667	1.948	74.87	64	79	.95	5.031
4	0	undef	undef	16.99	60	78	.22	4.439

NOTE: C = String length; H_T = Topological entropy; D_L = Lyapunov dimensionality; H_S = Shannon entropy.

SOURCE: Guastello (2000). permission here?

that strings that appear at least twice should be inspected for interpretive value.

All three applications produced sets of orbits that had diagnostic value for interpreting the behaviors of the groups. The complexities of the final sets of sequences were dependent on the particular scoring protocols that were used. Guastello (2000) applied two different scoring schemes to the same conversation and found that different string lengths, entropy levels, and so forth resulted from the two coding schemes. Rapp and Korslund (2000) also observed that the results of a symbolic dynamics analysis would be predicated on the particular scoring protocols that were used. It would appear, furthermore, that complex results would be improbable if the behavior coding systems were overly simplistic.

Conclusion

This chapter summarized three perspectives on nonlinear data analysis. Graphic techniques emanated from mathematical problems in which it was not possible to observe all the critical features of a function by simply inspecting a descriptive equation. They are potentially valuable in social science research as descriptive tools, but at present, they have little relevance to model building or hypothesis testing except, perhaps, in the crudest sense.

The mathematical concepts concerning dimensions are of primary importance for determining the nature of a dynamical process. If we know the dynamical process, then a great deal of variance over time can be explained. On the other hand, specific values of dimension are of lesser importance except in a comparative sense, such as healthy versus unhealthy systems.

The analytic habits of the natural sciences and social sciences vary sharply on the matter of filtering data before analysis, and it has been acknowledged that filtering distorts results. Analyses for the social sciences are now rooted in a relevant statistical theory that appears to make the filtering issue both a nonproblem and a nonrecommended procedure.

Nonlinear regression techniques are very versatile, and they can accommodate virtually any structural equation. Perhaps, the asset of versatility has become the limitation of the daunting number of possible models that a researcher might select for a hypothesis. The method of structural equations presented here limits the possibilities to hierarchical sets and should facilitate hypothesis formulation as it has done already.

The nonlinear regression approach can be expanded to encompass sinusoidal functions for the decomposition of limit cycles. Such problems are more frequently encountered in the biological sciences (Koyama, Yoneyama, Sawada,

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& Ohtomo, 1994) and economics (Puu, 1993) than they are in psychological problems. Symbolic dynamics techniques, such as the method of orbital decomposition, are also highly versatile for this purpose, especially where system states are observed rather than continuous numerical measurements.

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