

Transforming Primary QTS

Teaching Arithmetic in Primary Schools

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1. Getting your head around arithmetic

Learning Outcomes

By the end of this chapter you will:

- have a clearer understanding of what is meant by the terms ‘arithmetic’, ‘numeracy’ and ‘mathematics’, both in a historical context and in relation to this book;
- be aware of the past, current and likely future curricular requirements in relation to arithmetic;
- understand the key issues and challenges facing primary schools in relation to the teaching of arithmetic.

What is arithmetic?

A good starting point for a book about arithmetic is a consideration of what we understand it to mean. Before discussing the historical interpretations and the interpretation used throughout the rest of this book, let us first see what your thoughts are about arithmetic.

Activity

What is your definition of arithmetic? Think about it for a few minutes and jot down a few notes. What experiences and recollections, possibly from your childhood, have informed your view of what constitutes arithmetic?

There is a strong possibility that your definition relates to calculation, perhaps with a particular emphasis on mental arithmetic and traditional pencil and paper procedures with the numbers set out one underneath the other. You may even recall having a weekly mental arithmetic test when you were a child at school, although I wonder if you were actually taught how to calculate mentally. In my own experience children were expected to perform well in mental arithmetic tests, but were never taught the necessary skills and techniques. A parallel situation would be to expect someone to do a driving test without having had any driving lessons!

Arithmetic, mathematics and numeracy

Arithmetic is a word usually associated with bygone days and this would seem to be supported by quotations provided in the introductory section of the Cockcroft Report (DES, 1982, page xii). One of these originates from Her Majesty’s Inspectors in 1876, who state that *In arithmetic, I regret to say, worse results than ever before have been obtained*. Another is an extract from a Board of Education Report of 1925, which states that *Many have experienced some*

uneasiness about the condition of arithmetical knowledge and teaching at the present time. A third quotation is from a Mathematical Association report of 1954:

Experience shows that a large proportion of entrants [to trade courses] have forgotten how to deal with simple vulgar and decimal fractions, have very hazy ideas on some easy arithmetical processes, and retain no knowledge of algebra, graphs or geometry, if, in fact, they ever did possess any.

(DES, 1982, page xii)

If you would like to read more examples of the historical debate about mathematical education in the primary school, dip into a very interesting article by Alistair McIntosh (1981), details of which can be found in the Further Reading section at the end of this chapter.

The quotations presented above, perhaps in conjunction with your own reflections, lead to the conclusion that arithmetic relates to the mechanical processes of adding, subtracting, multiplying and dividing, either mentally or using pencil and paper. Importantly, this view of arithmetic is usually associated with a complete absence of any understanding of the underlying processes involved in its execution. Arithmetic was seen as a set of rules and procedures, which, if followed precisely, would yield correct answers, but often at the expense of much anxiety on the part of those executing them. Arithmetic in this sense is therefore a subset of mathematics, and indeed is not even a complete representation of what we usually refer to as number work.

The shift away from the widespread use of the word ‘arithmetic’ came about in the 1960s and 1970s with the advent of initiatives such as the Nuffield Mathematics Teaching Project from 1964 to 1971 and the publication of the Plowden Report in 1967. The widening of the mathematics curriculum and the move towards pupil-centred, as opposed to teacher-centred, education resulted in the expression ‘arithmetic’ no longer being an accurate representation of children’s experiences, and as a consequence it fell out of favour.

The expression ‘numeracy’ became popular in the 1990s with the introduction of the National Numeracy Project in September 1996. This pilot project, the forerunner of the National Numeracy Strategy, had a clear focus on the development of number skills and solving number problems, and so the first Numeracy Framework made no reference at all to areas of mathematics such as shape, space, measures or handling data. The definition of numeracy was modified further in The Final Report of the Numeracy Task Force, which presented it as being:

A proficiency that involves a confidence and competence with numbers and measures. It requires an understanding of the number system, a repertoire of computational skills and an inclination and ability to solve number problems in a variety of contexts. Numeracy also demands practical understanding of the ways in which information is gathered by counting and measuring, and is presented in graphs, diagrams, charts and tables.

(DfEE, 1998, page 11)

This much broader definition of numeracy, together with the subsequent publication of the Framework for Teaching Mathematics (DfEE, 1999), indicates a complete blurring of the boundaries between what is understood by the terms numeracy and mathematics; they had, in effect, come to mean the same thing, certainly in the primary phase, if not more widely.

With regard to the falling out of favour of the expression ‘arithmetic’, it should be noted that this word appears not even once in either the 1999 version or the 2006 version of the Framework for Teaching Mathematics. However, since the change of government in May 2010 and the demise of the National Strategies in March 2011, the word has acquired new found popularity, even if only among politicians. Shortly before the 2010 election, the shadow schools secretary, Michael Gove, stated in an interview to *The Times* newspaper that:

Most parents would rather their children had a traditional education, with children sitting in rows, learning the kings and queens of England, the great works of literature, proper mental arithmetic, algebra by the age of 11, modern foreign languages. That's the best training for the mind and that's how children will be able to compete.

(Thomson and Sylvester, 2010)

and more recently, government schools minister, Nick Gibb, has stated that:

Our reforms will give all pupils a solid grounding in reading and arithmetic, with the right catch-up support if they start to fall behind.

(Paton, 2011)

Current and future statutory requirements in relation to mathematics and arithmetic will be considered later in this chapter and indeed throughout this book, although for the time being I will leave you to ponder on what is meant by ‘proper mental arithmetic’.

One final observation with regard to the resurgence in the use of the word ‘arithmetic’: the word is not used at all in Ofsted’s 2008 and 2009 reports about mathematics, but occurs frequently in the 2011 report, although this is not surprising given that:

This survey was conducted following a ministerial request for Ofsted to provide evidence on effective practice in the teaching of early arithmetic.

(Ofsted, 2011, page 5)

This resurgence in the use of the word ‘arithmetic’ is the reason why it features in the title of this book, but in doing so, one of the aims is to encourage teachers to move away from the narrow interpretations typically associated with the word.

The scope of arithmetic as presented in this book

Unlike the definitions of arithmetic discussed above, this book will adopt a broader interpretation. Yes, it will focus on calculations involving the four arithmetical operations, but there will also be a strong emphasis on arithmetical understanding, as well as clear progression in the development of arithmetical techniques. This will begin with the recall of number facts in Chapter 2, followed by a detailed examination of mental arithmetic in Chapter 3, where a guiding principle will be that existing facts can be utilised flexibly in many different ways to mentally juggle with numbers. This flexible approach to arithmetic, which will depend on the numbers involved as well as personal choice, could not be further removed from the notion of blindly following memorised rules and procedures, as is the case with the traditional view of arithmetic. The same is true of the development of pencil and paper arithmetic presented in Chapters 4 and 5. Here, the rules that are the traditional algorithms, which depend on memory rather than understanding, are viewed as possible endpoints in children's arithmetical progression, not the starting points. The beginnings of pencil and paper arithmetic are therefore examined first in Chapter 4, building on the flexible mental methods presented earlier. Even when traditional pencil and paper arithmetic is introduced in Chapter 5 there continues to be an emphasis on understanding how these compact, efficient procedures actually work, so as to help you to move away from the notion of blindly following rules. Chapter 6 considers arithmetic involving fractions, decimals, percentages and ratios, and it makes use of the full range of arithmetical techniques discussed in Chapters 2 to 5. Finally, in Chapter 7, the vitally important role of technology is discussed, with a particular emphasis on calculators and spreadsheets.

Research Focus: Relational and instrumental understanding

Richard Skemp's seminal article, first published in 1977, presents two contrasting views of mathematical understanding: relational (*knowing both what to do and why*) and instrumental (*rules without reasons*), although it could be argued that the latter does not represent understanding at all. Examples of instrumental understanding include:

- the process of borrowing when using the traditional written method for subtraction;
- turning the fraction upside down and multiplying, when dividing by a fraction;
- taking a number across to the other side and changing the sign when solving equations;
- remembering that a minus and a minus is a plus when dealing with negative numbers.

→

Skemp argues that teachers and children will differ in their goals with regard to mathematical understanding. This can cause particular difficulties if the teacher is striving for relational understanding but the child is aiming for instrumental understanding ('Just tell me the rule!'). Likewise, there will be a similar conflict if the mathematical understanding goals are reversed, that is, the teacher just wants to teach rules, but the child is striving for relational understanding.

Skemp also discusses the advantages and disadvantages of each type of understanding. For example, instrumental mathematics is easier to grasp, it can be taught quickly and the rewards can be reaped almost immediately. In other words, you can learn the method and get a page of correct answers in no time at all. It is for these reasons, combined with the pressure of exams and getting through the syllabus, that many teachers choose instrumental understanding as the goal for their children.

Regarding the limitations of instrumental understanding Skemp describes a scenario in which two people are visiting an unfamiliar town. One has separate detailed sets of instructions to get to and from various locations in the town. The other has explored the town, familiarised himself with the roads and built up a mental map of where everything is. The first person is all right as long as he follows the instructions precisely, but if at any time he takes a wrong turn he will be lost and will remain lost until he retraces his steps and starts again. In contrast, the second person's mental map provides him with an infinite number of possibilities which will allow him to get from any starting point to any finishing point and, as Skemp states:

If he does take a wrong turn, he will still know where he is, and thereby be able to correct his mistake without getting lost; even perhaps to learn from it.

(1997, page 22)

If you are unable to obtain a copy of Skemp's article, instead read Chapter 1 of O'Sullivan *et al.* (2005), which presents and discusses a large extract from the original.

Curricular requirements for arithmetic

Before presenting the government's current priorities for arithmetic, it is worth examining the statutory requirements as they have developed over the last 25 years.

Recall of number facts

In terms of statutory National Curriculum requirements since 1989 there has been a consistent expectation with regard to children's recall of number facts. All versions of the programmes of

study have indicated that children, by the time they leave primary school, should be able to recall addition and subtraction facts for numbers up to 20 and multiplication facts up to 10×10 (DES, 1989; DfEE/QCA, 1999).

Mental arithmetic

The first version of the National Curriculum (DES, 1989) required children to:

- add or subtract mentally two two-digit numbers;
- add mentally several single-digit numbers.

The new National Curriculum, implemented from September 2000 (DfEE/QCA, 1999), provided more detail with regard to mental arithmetic, with children being expected to:

- add or subtract any pair of two-digit whole numbers;
- cope with particular cases of three-digit and four-digit additions and subtractions (for example $3000 - 1997$);
- double and halve any two-digit number;
- multiply and divide, initially in the range 1 to 100 (i.e. multiply or divide a two-digit number by a single digit number) and then for particular cases of larger numbers.

The National Curriculum programmes of study have always had to maintain a delicate balancing act, whereby sufficient detail is provided to ensure everyone is clear about the expectations, while at the same time producing a document which is concise and manageable for teachers. It is therefore impractical to provide more detail and exemplification than is summarised above, although the framework documents published by the National Numeracy Strategy and the Primary Strategies were able to accomplish this. Bearing this in mind, and also acknowledging that the programme of study represents only a minimum entitlement for children, the curriculum requirements listed above appear to be placing a sufficiently strong emphasis on mental arithmetic.

Pencil and paper arithmetic

The first National Curriculum (DES, 1989) required children to:

- use non-calculator methods to add and subtract two three-digit numbers;
- use non-calculator methods to multiply a three-digit number by a two-digit number and divide a three-digit number by a two-digit number.

As with mental arithmetic, the later version (DfEE/QCA, 1999) provided greater detail, with children being expected to:

- use written methods to add and subtract positive integers less than 1000, then up to 10,000;
- use written methods to add and subtract numbers involving decimals;

- use written methods for short multiplication and division to multiply and divide two-digit, three-digit and four-digit numbers by a single digit;
- use written methods for short multiplication and division to multiply and divide decimal numbers by a single digit;
- use written methods for long multiplication to multiply a three-digit number by a two-digit number;
- extend division to informal methods for dividing by a two-digit number.

It is interesting to note the subtle change of language from the first to the second version of the National Curriculum, with ‘non-calculator’ methods being replaced by ‘written methods’. It could be argued that the first version seems to be promoting informal written approaches more than the second. Expressions such as ‘written methods for long multiplication’ conjure up images of the traditional approach, with the digits arranged in columns in anticipation of the ‘carrying’ that is likely to follow. However, another interpretation is that given there is no specific mention of ‘traditional’ or ‘compact’ written methods, this provides the opportunity for teachers to introduce informal methods which initially build on children’s mental skills and bear no resemblance to the traditional approaches which will be encountered later.

The curriculum that never was

Prior to the change of government in May 2010, teachers were preparing for a new National Curriculum to be introduced in September 2011. The new programme of study for mathematics (QCDA, 2010) indicated that children should be taught to:

- know multiplication facts to 10×10 ;
- develop a range of strategies for calculating and checking (this includes mental methods, informal and formal written methods and using technology).

This represented a reduction in the level of detail presented in earlier versions, therefore providing greater scope when interpreting the requirements. The other notable feature is the more explicit acknowledgement of informal written methods.

Looking towards the future

Early in 2012 the government gave some indication of its current priorities for arithmetic, as follows.

- *Children must be able to recall quickly and accurately basic ‘number facts’ (e.g. number bonds and multiplication tables).*
- *Children must be fluent in applying quick, efficient written methods of calculation.*
- *Children need to be moved on rapidly to use appropriate efficient written methods, rather than becoming dependent on intermediate methods. This should be reflected*

in school policy. No child should be labouring with interim calculation methods in the long term.

- *Children need to understand and use the mathematical concepts that underpin number and arithmetic, such as place value and proportion. Without such concepts, pupils are ultimately less able to perform mental or written calculations, solve problems and reason mathematically.*
- *All children must leave primary school both proficient in the school's arithmetical algorithm for each operation and with a good understanding of the underpinning mathematics, both of which will equip them for solving unfamiliar problems and as a foundation for the more complex mathematics they will be taught in secondary school.*

(DfE, 2012a)

These priorities will be discussed in more detail at appropriate points throughout this book, although at this stage it is worth making the important point that there is no reference at all to mental calculation and only a passing acknowledgement of informal pencil and paper methods (referred to as 'intermediate' and 'interim' methods above).

In June 2012 the DfE published draft programmes of study for mathematics (DfE, 2012b) which provide more details of the proposed national curriculum which will be in place from September 2014. The draft programmes of study reflect the government priorities identified earlier, but additionally they do acknowledge the importance of mental arithmetic across the whole of the primary age range. However, there is minimal reference to informal pencil and paper methods. Instead the terms 'reliable' and 'efficient' written methods are used occasionally, but the expression 'formal written methods' is more predominant from Year 3 onwards. It is also noticeable that compared to the current and former versions of the national curriculum, traditional pencil and paper arithmetic in vertical columns is introduced at a much earlier age.

Both the March 2012 government priorities and the June 2012 draft programmes of study indicate that schools will be given a degree of flexibility when devising and implementing the mathematics curriculum, including their policies for arithmetic. In doing so, it is hoped that the wealth of research evidence clearly demonstrating the crucial role of approaches other than formal written methods, is acknowledged by schools. Thus a broader and more balanced interpretation of arithmetic, in tune with the one presented in this book, will inform classroom practice.

Children's attainment in arithmetic

Attempts to compare children's levels of attainment across different time periods are fraught with the same difficulties as when making comparisons of sporting excellence through the ages. Who is the greatest goal-scorer the footballing world has ever seen? On current form you might

say Lionel Messi, although a few years ago perhaps Thierry Henry. Go further back in time and it might be Marco van Basten, or Diego Maradona, or George Best, or Pele, or Ferenc Puskás. The simple fact is that it is impossible to make meaningful comparisons, because these cannot possibly take account of the context in which each individual player was excelling. The same is true of pupil attainment. It is therefore a complete waste of time and effort to attempt to compare standards now with those in the 1990s or before. Even during a period when, supposedly, the same instruments have been used to measure attainment, the comparisons need to be viewed with caution. The proportion of children achieving the highest grades in GCSE mathematics has risen steadily over the years, but there are many who would argue that it is far easier to be awarded a grade C now than it was in the 1980s. Similarly with Key Stage 2 National Tests results, the percentage of children achieving level 4 or above has risen from 44 per cent in 1995 to 80 per cent in 2011, and yet there have been constant cries throughout this period proclaiming that standards need to be raised further, or that school leavers lack basic skills, or that pupils in other countries perform better than ours in international tests, or that the quality of teaching needs to be improved. There is certainly no room for complacency, but I would suggest that things are sometimes not as bad as the headlines would suggest, and it is a fact of life that people look back to bygone days through rose-tinted spectacles and so are perpetually critical of their current lot. Cockcroft (DES, 1982, page xiii) recognised this thirty years ago when he observed that *criticism of mathematics teaching is not new* and it is likely that he would reach the same conclusion today.

Current issues and challenges

Despite the cautionary words above, it is appropriate to identify the issues that need to be addressed if we want to improve the overall quality of children's arithmetical capability in the primary school. The following list focuses specifically on issues relating to arithmetic, rather than broader mathematical concerns, and is based on several recent publications from Ofsted, as well as other government sources.

- It is essential that children are able to recall number facts, such as multiplication tables and number bonds, because these, together with an understanding of place value, lay the foundations for mental and pencil and paper arithmetic (DfES, 2007; DCSF, 2007; Ofsted, 2011).
- Greater emphasis should be given to the development of children's mental arithmetic skills and this must be maintained throughout the primary years, even after written methods have been introduced (DfES, 2007; DCSF, 2007; DCSF, 2008).
- Children should not use written or calculator methods for calculations that can be done mentally (DfES, 2007; DCSF, 2007; Ofsted, 2008).
- Arithmetical understanding should be developed through high-quality questioning and discussion, rather than simply presenting rules and procedures to be memorised (DCSF, 2007; DCSF, 2008; Ofsted, 2008).

- Individual arithmetical methods and operations should not be considered in isolation. Instead, children need to be made aware of the connections and relationships that exist between the methods they are learning and between the four arithmetical operations (DCSF, 2007; DCSF, 2008; Ofsted, 2008; Ofsted, 2011).
- Greater use needs to be made of practical resources and visual aids to support the teaching of arithmetic (DfES, 2007; DCSF, 2007; DCSF, 2008; Ofsted, 2008; Ofsted, 2009; Ofsted, 2011).
- Children must be taught how to use calculators more effectively (DCSF, 2007; Ofsted, 2011).
- More opportunities need to be provided to allow children to use and apply their arithmetical skills through problem solving and open-ended, investigative tasks, as well as through cross-curricular activities (DfES, 2007; DCSF, 2007; DCSF, 2008; Ofsted, 2008; Ofsted, 2009; Ofsted, 2011).
- Teachers need to have the necessary subject and pedagogical knowledge to address the issues listed above (DCSF, 2008; Ofsted, 2008; Ofsted, 2009; Ofsted, 2011).

The remaining chapters of this book aim to address the final bullet point above, so that you will be suitably equipped to meet the challenges you face in teaching arithmetic in the primary school.

Learning Outcomes Review

It is hoped that this introductory chapter has set the scene for you in terms of what is meant by arithmetic and the extent to which it features in the current and likely future curriculum requirements. The historical perspectives presented should also have contributed to your overall understanding of the issues surrounding the teaching of arithmetic. However, the most important aspect of this chapter is that it has identified for you the key challenges facing schools and teachers if they want to develop children's arithmetical capability. The remainder of this book will help you to meet these challenges.

Further Reading

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