

**CHAPTER 5**

# Standardized Scores—Do You Measure Up?

So far, we've been working with what is called *raw data*. From this raw data or raw scores, we have been able to create frequency distributions and frequency curves and to examine measures of central tendency and of dispersion. These concepts are foundational to testing and measurement. It's time, however, to start learning more about scores themselves.

Often we want to compare people whom we assess with people who are more representative of others in general. Raw scores won't allow us to do this. With raw scores we can only compare the people within the same group who took the same test. We can't compare their scores to people who were not in the group that we tested. This is a dilemma! So what do we do?

Guess what?! You transform your raw scores to standard scores. When we standardize scores, we can compare scores for different groups of people and we can compare scores on different tests. This chapter will reveal the secrets of four different standard scores: *Percentiles*, *Z scores*, *T scores*, and *IQ scores*. Aren't you glad? (If you aren't glad now, you will be when you take your next exam.)

## Percentiles—What They Mean in Measurement

Are you ready to begin? The first type of standard score is a percentile score. A *percentile* score, more commonly referred to as a *percentile rank*, represents the percentage of people in a group who scored at or below any given raw score. Crystal clear? Yeah, that sounds like gobbledygook to us too. So, maybe this example will make it clearer.

Let's start with a formula. Mathematically, a percentile rank (*PR*) equals the cumulative frequency (*cf*) that corresponds to a given (*X*) score (in this

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case, the one for which you are trying to find the percentile rank) divided by the number ( $N$ ) of people in the frequency distribution and then multiplied by 100. The formula looks like this:

$$PR = \frac{cf}{N} \times 100$$

Figuring the  $PR$  is really simple and straightforward. So let's do it with the following data set:

$X$	$f$	$cf$
10	1	10
8	1	9
7	2	8
5	1	6
2	3	5
1	2	2

We want to know the percentile rank for a score of 2. Using the data above, what is the  $cf$  for a score of 2? THAT'S RIGHT—5. What is your sample size or  $N$ ? RIGHT AGAIN—10. Good for you—you haven't forgotten how to read a cumulative frequency distribution. Now, let's plug these into our formula:

$$PR = \frac{cf}{N} \times 100$$

$$PR = \frac{5}{10} \times 100$$

$$PR = .5 \times 100$$

$$PR = 50$$

This  $PR$  of 50 means that 50% of the people who took this test scored 2 or below. (Gosh, they didn't do very well.)

Let's find the  $PR$  for the score of 8 using the same data set. The  $N$  has not changed. It is still 10. However, the  $cf$  has changed. It is now 9. When we plug this information into our formula, we get the following:

$$PR = \frac{cf}{N} \times 100$$

$$PR = \frac{9}{10} \times 100$$

$$PR = .9 \times 100$$

$$PR = 90$$

Anyone who scored an 8 on this test has a *PR* of 90 and scored the same as or better than 90% of the people who took this test. Pretty good, wouldn't you say?!

There are two things that we really want to make sure you understand.

**Thing One:** A percentile rank is not the same as a percentage, although they are closely related. The percentile rank tells us the rank of a score with respect to the group that the score is in. When you put a percentage sign with your percentile rank, you know the percentage of people who score at or below a raw score that corresponds to that percentile rank.

**Thing Two:** We talk about “scoring the same as or better than” a certain percentage of people who took the test. We do not say just “better than.” (Some authors use the latter, but in measurement it's more accurate to think about a percentile rank as reflecting scores that are “the same as or better than.”)

## Percentile Ranks for Grouped Data

Although a similar process is used to calculate percentile rank for grouped data, more steps are involved. Don't let the formula scare you. We'll lead you through it bit by bit. It is very important for you to understand that when you have grouped (or interval) data, you are trying to determine the proportion of distance you will need to go into an interval in order to find the *PR* of a score that falls in the interval. To do this, these are the variables you need to be able to identify:

- $X$  = raw score
- $N$  = total number of people in the group
- $i$  = interval width
- $f_x$  = number of people with scores in the interval containing  $X$
- $cf_b$  = *cf* of the interval below the interval containing  $X$
- $LL$  = lower limit of the interval containing  $X$ . (Remember *LL* from Chapter 2.)

The formula is

$$PR = \frac{[cf_b + f_x \left[ \frac{X - LL}{i} \right]] \times 100}{N}$$

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The formula may look overwhelming, but if you take it slowly and work from inside the first parentheses out, it's a piece of cake. The trick is doing it in little steps. Are you thinking, "OK, if you think it's so easy, you do it!"? OK, we will.

Here is a new set of grouped data. For this data set, what is the *PR* for a score of 21?

<i>Interval</i>	<i>f</i>	<i>cf</i>
25–29	5	30
20–24	5	25
15–19	10	20
10–14	2	10
5–9	5	8
0–4	3	3

First, we need to define each component of the equation.

- $X = 21$
- $N = 30$
- $i = 5$
- $f_x = 5$
- $cf_b = 20$
- $LL = 19.5$

Now, let's plug these values into the equation for *PR*.

$$PR = \frac{[cf_b + f_x[\frac{X-LL}{i}] \times 100]}{N}$$

$$PR = \frac{[20 + 5[\frac{21-19.5}{5}] \times 100]}{30}$$

$$PR = \frac{[20 + 5[\frac{1.5}{5}] \times 100]}{30}$$

$$PR = \frac{[20 + 5[.3] \times 100]}{30}$$

$$PR = \frac{(20 + 1.5) \times 100}{30}$$

$$PR = \frac{(21.5) \times 100}{30}$$

$$PR = .7167 \times 100$$

$$PR = 71.67$$

The *PR* for a score of 21 for this set of data is 71.67. This means that anyone who scored 21 scored the same as or better than 71.67% of the people in this data set.



### Let's Check Your Understanding

Why don't you try it now? What is the  $PR$  for a score of 14 and what does this mean?

1. Find the values for the following components of the equation:

- $X =$
- $N =$
- $i =$
- $f_x =$
- $cf_b =$
- $LL =$

2. Plug these values into the equation for  $PR$ .

$$PR = \frac{[cf_b + f_x \left[ \frac{X - LL}{i} \right] \times 100]}{N}$$

$PR =$

$PR =$

$PR =$

$PR =$

$PR =$

$PR =$

Hint:  $PR = 32.67$ . If you didn't get this answer, check out what we did below.

3. What does this  $PR$  tell you?

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### Our Model Answers

1. The values for the components of the equation are:

- $X = 14$
- $N = 30$
- $i = 5$
- $f_x = 2$
- $cf_b = 8$
- $LL = 9.5$

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2. The completed equation is

$$PR = \frac{[cf_b + f_x \left[ \frac{X - LL}{i} \right] \times 100]}{N}$$

$$PR = \frac{[8 + 2 \left[ \frac{14 - 9.5}{5} \right] \times 100]}{30}$$

$$PR = \frac{[8 + 2 \left[ \frac{4.5}{5} \right] \times 100]}{30}$$

$$PR = \frac{[8 + 2[.9] \times 100]}{30}$$

$$PR = \frac{(8 + 1.8) \times 100}{30}$$

$$PR = \frac{(9.8) \times 100}{30}$$

$$PR = .3267 \times 100$$

$$PR = 32.67$$

3. What does this *PR* tell you?

**A person who scores 14 has a *PR* of 32.67. This means he or she scored the same as or better than 32.67% of the people who took this test.**

## Some Final Thoughts About Percentile Ranks

Another neat thing about percentile ranks is that you can also figure out what percentage of people or scores fell between two scores. So, if we asked you what percentage of people scored between 14 and 21, you now know everything you need to know to answer. Just subtract the *PR* for the score of 14 ( $PR_{14} = 32.67$ ) from the *PR* for the score of 21 ( $PR_{21} = 71.67$ ). Voila! ( $PR_{21} - PR_{14} = 71.67 - 32.67 = 39$ , which means that 39% of the people scored between scores of 14 and 21.) You can even go one step further and tell us the exact number of people who make up this 39% by multiplying the 0.39 by the number of people (*N*) who took the test ( $0.39 \times N = 0.39 \times 30 = 11.7$ ). Therefore, you could tell us that 11.7 people scored between 14 and 21. We know, there's no such thing as a 0.7 person, but this is the mathematical answer so we'll live with it.

The beauty of a percentile rank and its accompanying percentile value is that it allows you to make accurate comparisons about people who took the

same test and to draw conclusions about any one person in that group. It's also very convenient that you can use simple or grouped data and still draw accurate conclusions.

Remember that a *standard score* is a transformed raw score. A percentile rank is just one type of a standard score. There are other types of standard scores that are very important in measurement.

## Z Scores

The foundational standard score in measurement is the *Z score*. A *Z score* is based on the normal, bell-shaped curve and is formed from deviation scores. Remember deviation scores from the last chapter? A deviation score is the difference between any one score and the mean ( $X - M$ ). If this deviation score is divided by the standard deviation (*SD*) for that group of scores, we have transformed the raw score into a *Z score*. The formula for a *Z score* is:

$$Z = \frac{X - M}{SD}$$

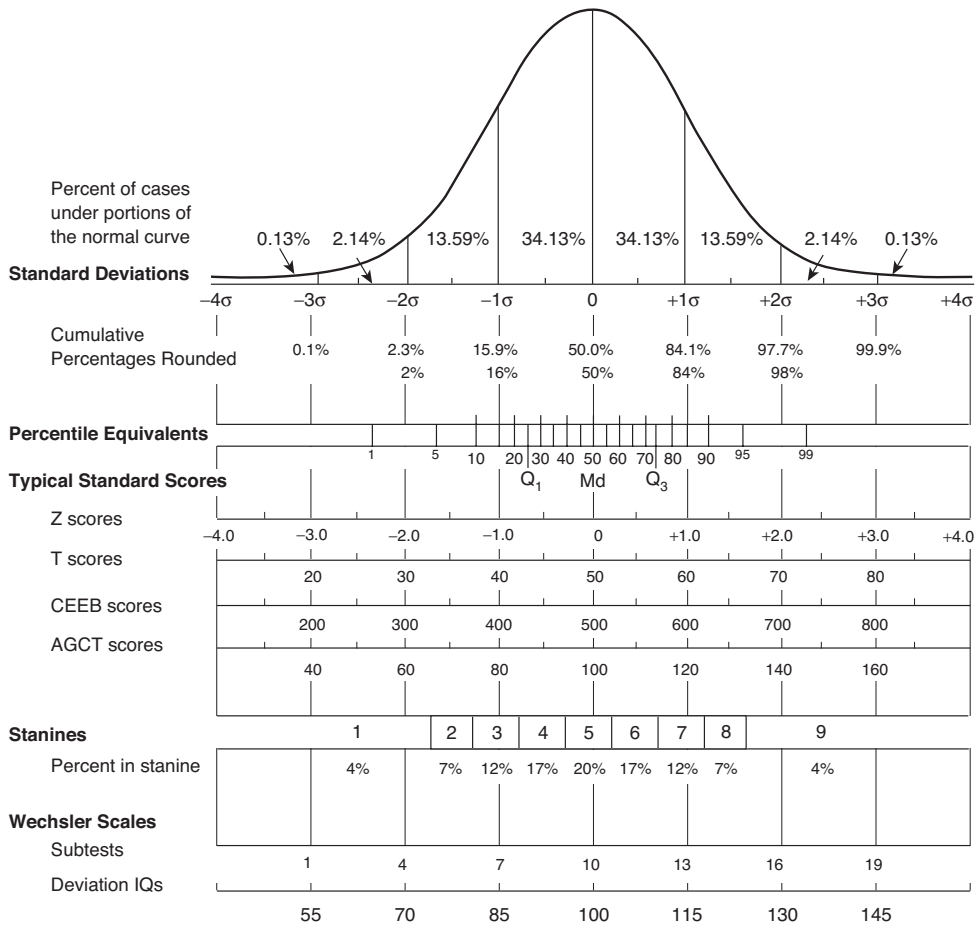
A *Z score* is the deviation of a score from the mean expressed in standard deviation units. The sum of the *Z scores* is always zero ( $\Sigma Z = 0$ ). The mean for a curve of *Z scores* is also always 0, and the *SD* is always 1. That means that the variance ( $s^2$ ) also equals 1. The *Z scores* below the mean have a negative value and those above the mean have a positive value. *Z scores* range from  $-4.00$  to  $+4.00$ . The convention is to present *Z scores* with two decimal places. There is also a wonderful, helpful *Z-score* table that can be found in most statistics books. We placed a *Z score* table in Appendix A.

## Why Transform a Raw Score to a Z Score?

The value of transforming a raw score to a *Z score* is twofold.

**Value One:** Any score can be expressed as a percentile rank by referring its *Z score* to the normal, bell-shaped curve (see Figure 5.1). As you do this, you need to remember that between the mean and  $1SD$  you have 34.13% of all scores, between 1 and  $2SD$  you have 13.58% of all scores, and between  $2SD$  and  $3SD$  you have 2.14% of all scores. This is true whether you're looking at the portion of the curve above the mean or below it.

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**Figure 5.1** Z Scores and the Normal Curve

NOTE: This chart cannot be used to equate scores on one test to scores on another test. For example, both 600 on the CEEB and 120 on the AGCT are one standard deviation above their respective means, but they do not represent “equal” standings because the scores were obtained from different groups.

A  $Z$  of 0 is always located at the mean, which indicates that 50% of the scores fall below it and 50% of the scores fall above it. If someone’s  $Z$  score is  $+1.00$ , it is  $1SD$  above the mean. To calculate the percentile rank for a  $Z$  score of  $+1.00$ , we need to find the percentage of scores from the bottom point of the curve up to the  $Z$  score of  $+1.00$ . We do this by adding 34.13%, the percentage of scores between the mean and  $1SD$  above the mean, to 50%, the percentage of all scores that fall below the mean.

$$+1Z = 34.13\% + 50\% = 84.13\%$$

We now know that the person who had a  $Z$  score of  $+1.00$  scored the same as or better than 84.13% of the people who took this test. So this person’s percentile rank is 84.13.



Now, if a person's  $Z$  score is  $-1.00$ , it is  $1SD$  below the mean. To calculate the percentile rank for a  $Z$  score of  $-1.00$ , we need to find the percentage of scores from the bottom point of the curve up to the  $Z$  score of  $-1.00$ . We do this by subtracting  $34.13\%$ , the percentage of scores between  $-1SD$  and the mean, from  $50\%$ , the percentage of scores that fall below the mean.

$$-1Z = 50\% - 34.13\% = 15.87\%$$

So, we know that the person who had a  $Z$  score of  $-1.00$  scored the same as or better than  $15.87\%$  of the people who took this test. So this person's percentile rank is  $15.87$ .

**Value Two:** The second value of transforming raw scores to  $Z$  scores is that we can compare one person's score on one test with his or her score on another test. Some people try to compare raw scores, but they are wrong, wrong, wrong! You can only make comparisons across tests when you have standardized both sets of test scores.

For example, you're the principal working at Martin Luther King Junior High School and you receive a phone call from the parents of one of your students, Simon. They are worried about the test papers he just brought home from his math and his English classes. His math test score was  $48$ . His English test score was  $58$ . His parents want to know if they should hire tutors to help Simon.

If you simply looked at his two raw scores, you might erroneously think he is doing better in English by his score of  $58$  than he is doing in math by his score of  $48$ . This would be a mistake since these two tests have different means and standard deviations. Being the measurement scholar that you are, you know that you cannot compare raw scores, such as the ones Simon's parents gave you. Wisely, you consult his math and his English teachers to find out the class means and standard deviations on each of these tests. (Lucky you, these teachers were conscientious and had calculated these statistics.) The class math test mean was  $40$  and the  $SD$  was  $8$ . The class English mean was  $62$  and the  $SD$  was  $4$ . Because you know you can't compare raw scores, you standardize Simon's two test scores by using the handy-dandy  $Z$  score formula.

Simon's math test: $X = 48$	Simon's English test: $X = 58$
Class $M = 40$	Class $M = 62$
$SD = 8$	$SD = 4$
$Z = \frac{(X - M)}{SD}$	$Z = \frac{(X - M)}{SD}$
$Z = (48 - 40)/8$	$Z = (58 - 62)/4$
$Z = 1.00$	$Z = -1.00$

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Even though Simon's raw score in English is higher than his raw score in math, you can tell that Simon did much better on his math test than on his English test just by eyeballing the standardized  $Z$  scores. Simon scored at the 84.13th percentile in math compared to his classmates. His percentile rank equals the percent of those who scored below the mean (50%) plus the percent of those who scored from the mean to  $+1Z$  above the mean ( $PR_{+1Z} = 50\% + 34.13\% = 84.13\%$ ) or 84.13th percentile. However, he scored only at the 15.87th percentile in English ( $PR_{-1Z} = 50\% - 34.13\% = 15.87\%$ ). Simon needs help in English. On this test he did better than only 15.87% of his classmates. You suggest to his parents that they should talk to his seventh-grade English teacher and maybe hire a tutor for English. Being a good principal, you also reassure Simon's parents that he seems to be doing well in math.

Let's do one more example. Remember our good friend Ryan who had complained about being stressed and not having any friends. He had taken both the College Stress Scale (CSS) and the Social Support Inventory (SSI). On the CSS, Ryan had scored 92. The mean for college freshmen was 71.78 with a  $SD$  of 16.76. On the SSI, Ryan had scored 15. The mean for college freshmen was 21.65 with a  $SD$  of 4.55. As Ryan's counselor, you are trying to decide whether to first address his loneliness or his stress. To help you make this decision, you want to compare Ryan's level of stress with his level of loneliness. To do this you have to transform his raw scores to  $Z$  scores.

Ryan's CSS score: $X = 92$	Ryan's SSI score: $X = 15$
Freshmen $M = 71.78$	Freshmen $M = 21.65$
$SD = 16.76$	$SD = 4.55$
$Z = \frac{(X - M)}{SD}$	$Z = \frac{(X - M)}{SD}$
$Z = (92 - 71.78)/16.76$	$Z = (15 - 21.65)/4.55$
$Z = +1.21$	$Z = -1.46$

If you were Ryan's counselor, you might conclude that he is both stressed (with a  $Z$  more than  $1SD$  above the mean) and has little social support (with a  $Z$  almost  $1.5SD$  below the mean). Clearly, he needs to learn how to make friends to build his support network. He could also benefit from some stress management training. You decide to focus on making friends while giving him some stress management techniques.



### Let's Check Your Understanding

One of Ryan's two friends, Paula, also took the CSS and the SSI. What are the  $Z$  scores for each of Paula's test scores and how would you interpret them?

Paula's CSS score: $X = 75$	Paula's SSI score: $X = 26$
Freshmen $M = 71.78$	Freshmen $M = 21.65$
$SD = 16.76$	$SD = 4.55$
$Z = \frac{(X - M)}{SD}$	$Z = \frac{(X - M)}{SD}$
$Z =$	$Z =$
$Z =$	$Z =$

1. What does Paula's stress  $Z$  score tell us about her level of stress?

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2. What does Paula's social support  $Z$  score tell us about her level of support?

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3. Compare Paula's level of stress and social support.

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4. Why would looking at her raw scores mislead you?

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### Our Model Answers

We've completed the table below with the correct  $Z$  scores. How did you do?

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Paula's CSS score: $X = 75$	Paula's SSI score: $X = 26$
Freshmen $M = 71.78$	Freshmen $M = 21.65$
$SD = 16.76$	$SD = 4.55$
$Z = \frac{(X - M)}{SD}$	$Z = \frac{(X - M)}{SD}$
$Z = \frac{(75 - 71.78)}{16.76}$	$Z = \frac{(26 - 21.65)}{4.55}$
$Z = +0.19$	$Z = +0.96$

1. What does Paula's stress  $Z$  score tell us about her level of stress?  
**Paula's  $Z$  score is only +0.19, which is very close to the  $Z$  mean of 0. Paula seems to be perfectly average in her level of stress compared to other freshmen.**
2. What does Paula's social support  $Z$  score tell us about her level of support?  
**Paula's  $Z$  score is almost 1SD above the freshmen mean for social support. We can conclude that she has high average social support.**
3. Compare Paula's level of stress and social support.  
**Paula's  $Z$  scores fall within 1SD above the mean. This means that she is like most other freshmen (at least 68.26% of them). If we were to be very picky, we would also be able to conclude that she is almost above average in perceived social support and just about perfectly average in her experienced stress.**
4. Why would looking at her raw scores mislead you?  
**If we just looked at her score of 75 for stress and her score of 26 for social support, we might have concluded that she has more stress than support. How wrong we would be!!!**

You are doing such a good job that we've decided to introduce you to the  $Z$  table that we've placed in Appendix A. Some brilliant mathematician (we don't know his or her name) calculated the exact percentile score to correspond with every possible  $Z$  value. For example, Ryan's  $Z$  score of +1.21 on the CSS means that he scored the same or higher than 88.69% of other freshmen on stress. We arrived at this value by looking at the  $Z$  value of +1.21 in the  $Z$  table in Appendix A and saw that this value reflected 38.69% of the area between the mean and 1.21SD above the mean. Because we wanted the percentile score, we could not ignore the 50% who scored below the  $Z$  mean of 0. When we added this 50% to the 38.69%, we know that Ryan scored at the 88.69th percentile.

For his social support  $Z$  score of  $-1.46$ , he scored the same as or higher than only 7.21% of other freshmen. We arrived at this value by looking at the  $Z$  value of  $-1.46$  in the  $Z$  table in Appendix A and saw that this value represented 7.21% of the area below the  $Z$  score of  $-1.46$ . Therefore, we know that Ryan scored at the 7.21st percentile.



### Now It's Your Turn

It's time for you to find the exact percentile ranks for Paula. Remember that her  $Z$  scores were  $+0.19$  on the CSS and  $+0.96$  on the SSI. OK, what are her percentile ranks?

1. CSS  $Z$  of  $+0.19$  =

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2. SSI  $Z$  of  $+0.96$  =

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(Hint: Don't forget to add the 50% who scored below the mean.)



### Our Model Answers

1. CSS  $Z$  of  $+0.19$  =

The percentile rank for a  $Z$  score of  $+0.19$  equals **57.53**. We arrived at this value by looking at the  $Z$  value of  $+0.19$  in Appendix A and found that this value represents **7.53%** of the area between the mean and  $Z$  score. Because the  $Z$  value is positive, we have to add the **50%** that represents the scores below the mean to the **7.53%** and arrive at **57.53%** of scores at or below a  $Z$  score of  $+0.19$ . Therefore, the percentile rank is **57.53**.

2. SSI  $Z$  of  $+0.96$  =

The percentile rank for a  $Z$  score of  $+0.96$  equals **83.15**. We arrived at this value by looking at the  $Z$  value of  $+0.96$  in Appendix A and found that this value represents **33.15%** of the area between the mean and  $Z$  score. Because the  $Z$  value is positive, we again have to add the **50%** that represents the scores below the mean to the **33.15%** and arrive at **83.15%** of scores at or below a  $Z$  score of  $+0.96$ . Therefore, the percentile rank is **83.15**.

## Other Standard Scores

Two other types of standard scores frequently used in measurement are  $T$  scores and  $IQ$  scores. Some of you may also be familiar with ACT, SAT, and

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GRE scores, which are also standardized scores. For example, both the SAT and GRE have a set mean of 500 and a *SD* of 100. Don't groan, your GRE Verbal of 490 wasn't that bad. You are rubbing elbows (literally and figuratively) with the average person who took the GRE.

### ***T* Scores**

A frequently used standard score is the *T score*. *T* scores, often reported for personality inventories, have a mean of 50 and a standard deviation of 10. A *T* score is computed by multiplying the *Z* score by 10 and adding 50.

$$T = 10(Z) + 50$$

For Paula's stress *Z* score of +0.19, the equivalent *T* score is 51.9. We got this by doing the following:

$$T = 10(Z) + 50$$

$$T = 10(0.19) + 50$$

$$T = 1.9 + 50$$

$$T = 51.9$$

The *T* score becomes particularly useful when you are creating profiles of an individual. Two popular personality inventories, the *Personality Research Form* (PRF) and the *California Personality Inventory*, report results as *T* scores. Let's say on the PRF you score 40 ( $-1SD$  below the mean) and I score 60 ( $+1SD$  above the mean) on Dominance. My score is  $2SD$  above yours. If I were you, I wouldn't get into an argument with me, because I have a much stronger need to dominate and win than you do. By knowing my *T* score, you know something about my personality and the strength of my personality characteristics in comparison to a norm group. (We will explain norm groups in Chapter 6.)

### ***IQ* Scores**

A second frequently used standard score is the *IQ score*. Like *T* scores, *IQ* scores are derived from *Z* scores. (You probably didn't realize how valuable the *Z* score was going to be to your life.) The formula for converting a *Z* score to an *IQ* score is

$$IQ = SD(Z) + M$$

In this formula, you need to know the *SD* and *M* for the *IQ* test and the person's *Z* score.

Many intelligence tests use IQ scores that have a mean of 100 and a standard deviation of 15 or 16. For example, each of the Wechsler tests of intelligence has a mean of 100 and a *SD* of 15. The Binet IV has a mean of 100 and a *SD* of 16. For the Wechsler scales the formula for an IQ score is:

$$\text{IQ} = 15(Z) + 100$$

The formula for an IQ score using the Binet IV would be:

$$\text{IQ} = 16(Z) + 100$$

Most of the time you will not have to calculate a *T* score or an IQ score. Most assessment instruments will report results in standard scores or provide you with a table to convert raw scores directly to standard scores without having to calculate the *Z* score first. What you'll need to know, however, is how to compare and interpret these standard scores.

Let's look at an example. Simon's 15-year-old sister, Sarah, has been referred to the school psychologist for cutting classes. Sarah takes a series of tests including the Wechsler Intelligence Scale for Children, Fourth Edition (WISC-IV) and the PRF. Her IQ score on the WISC-IV is 133 and her *T* scores on the PRF are 39 for Harmavoidance, 62 for Play, 60 for Impulsivity, and 63 for Exhibitionism. Based on Sarah's IQ score of 133, it is evident that she is very bright. (Surely you saw that she was more than 2SD above the mean of 100.) Each of her scores on the PRF scales is one or more standard deviations away from the mean, which signifies that she tends to be extreme on these scales that happen to assess "at-risk" tendencies. She is a teenager who likes to have a good time (Play), doesn't think before she acts (Impulsivity), likes to be the center of attention among her friends (Exhibitionism), and is not careful in situations that could be harmful (Harmavoidance). Sarah needs to look at how these personality characteristics get her into trouble and how she can use her intelligence to help her make better decisions about her behaviors.

## Key Terms

Check your understanding of the material by explaining the following concepts. If you aren't sure, look back and reread.

- Percentile rank
- *Z* score
- *T* score
- IQ score

## Models and Self-instructional Exercises

### Our Model

You may not know this, but when Gandalf (from *Lord of the Rings*) was selecting which Hobbits to help him fight the Dark Lord Sauron, he gave the Hobbits tests to measure courage, endurance, and power-hungriness. The Hobbits had a mean of 22 ( $SD = 4$ ) on the courage scale, a mean of 53 ( $SD = 7$ ) on the endurance scale, and a mean of 13 ( $SD = 2$ ) on the power-hungriness scale. Mr. Frodo scored 30 on courage, 60 on endurance, and 7 on power-hungriness. Samwise Gamgee scored 26 on courage, 62 on endurance, and 9 on power-hungriness. Whom should Gandalf pick?

To answer this question, we first want to calculate the  $Z$  scores for both Mr. Frodo and Samwise on each scale.

	Mr. Frodo	Samwise Gamgee
Courage	$Z = \frac{X - M}{SD}$ $Z = \frac{30 - 22}{4}$ $Z = \frac{8}{4} = +2.00$	$Z = \frac{X - M}{SD}$ $Z = \frac{26 - 22}{4}$ $Z = \frac{4}{4} = +1.00$
Endurance	$Z = \frac{X - M}{SD}$ $Z = \frac{60 - 53}{7}$ $Z = \frac{7}{7} = +1.00$	$Z = \frac{X - M}{SD}$ $Z = \frac{62 - 53}{7}$ $Z = \frac{9}{7} = +1.29$
Power-hungriness	$Z = \frac{X - M}{SD}$ $Z = \frac{7 - 13}{2}$ $Z = \frac{-5}{2} = -2.50$	$Z = \frac{X - M}{SD}$ $Z = \frac{9 - 13}{2}$ $Z = \frac{-4}{2} = -2.00$

If we compare Mr. Frodo and Samwise Gamgee on their courage scores, Mr. Frodo, with a  $Z$  score of +2.00, is more courageous than 97.72% of the Hobbits who took this test. Although Sam, with a  $Z$  score of +1.00, is more courageous than 84.13% of the Hobbits who took this test, he is not as courageous as Mr. Frodo. Indeed, 13.59% of the Hobbits scored between



Mr. Frodo and Sam ( $97.72\% - 84.13\% = 13.59\%$ ). To get these figures, we remembered our normal curve and the percentage of people at different standard deviation points. If you need to refresh your memory, reread the beginning of this chapter.

Now let's compare our two courageous Hobbits on endurance. Mr. Frodo had a  $Z$  score of  $+1.00$ , which means he scored higher than  $84.13\%$  of the Hobbits. Sam had a  $Z$  score of  $+1.29$ , which means he scored higher than  $90.15\%$  of the Hobbits (we used the  $Z$  table in Appendix A to find this percentage). Samwise Gamgee has higher endurance than Mr. Frodo, even though they are both high in endurance. Approximately  $6\%$  of the Hobbits scored between Mr. Frodo and Sam ( $90.15\% - 84.13\% = 6.02\%$ ).

Finally, we need to look at their power-hungriness scores. Mr. Frodo had a  $Z$  score of  $-2.50$ . His need for power is higher than only  $0.62\%$  of the Hobbits. Sam had a  $Z$  score of  $-2.00$ , which means that his need for power is higher than  $2.27\%$  of the Hobbits. Neither Mr. Frodo nor Samwise has a drive to gain personal power.

When Gandalf looks at all of these  $Z$  scores and percentile ranks, he selects Mr. Frodo to be the ring bearer and the story's main character. However, Sam's extra high endurance coupled with his courage and low power-hungriness make him a perfect companion for Mr. Frodo as they try to destroy the ring and the Dark Lord Sauron.



### Now It's Your Turn

Hey Dude!!! Do you remember Bill and Ted before they took their "excellent adventure"? They had just taken their midterms in history and political science. Bill, who hadn't been paying attention in either class, scored 58 in history and 65 in political science. Ted, who was no shining light, scored 55 in history and 68 in political science. Their class means were  $78(SD = 10)$  for history and  $82(SD = 8)$  for political science. Based on these data, complete this table and answer questions 1 through 4. Hint: You will need to find the  $Z$  scores and use the  $Z$  table in Appendix A to help you find percentile ranks.

1. What was Bill's percentile rank in history and what percent of his classmates did he score better than?

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2. What was Bill's percentile rank in political science and what percent of his classmates did he score better than?

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3. What was Ted's percentile rank in history and what percent of his classmates did he score better than?

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4. What was Ted's percentile rank in political science and what percent of his classmates did he score better than?

---

	<i>Bill</i>	<i>Ted</i>
History	$Z = \frac{X - M}{SD}$ $Z =$ $Z =$ $PR =$	$Z = \frac{X - M}{SD}$ $Z =$ $Z =$ $PR =$
Political Science	$Z = \frac{X - M}{SD}$ $Z =$ $Z =$ $PR =$	$Z = \frac{X - M}{SD}$ $Z =$ $Z =$ $PR =$

Their history teacher, Mr. Carlin, is trying to standardize the history scores so that students in different classes could be compared to each other. Mr. Carlin converts everyone's scores to *T* scores.

5. What is Bill's *T* score on his history exam?

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6. What is Ted's *T* score on his history exam?

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### Our Model Answers

To begin, we will first calculate the *Z* scores for both Bill and Ted for each class.

	Bill	Ted
History	$Z = \frac{X - M}{SD}$ $Z = \frac{58 - 78}{10}$ $Z = \frac{-20}{10} = -2.00$ $PR = 2.27$	$Z = \frac{X - M}{SD}$ $Z = \frac{55 - 78}{10}$ $Z = \frac{-23}{10} = -2.30$ $PR = 1.07$
Political Science	$Z = \frac{X - M}{SD}$ $Z = \frac{65 - 82}{8}$ $Z = \frac{-17}{8} = -2.13$ $PR = 1.66$	$Z = \frac{X - M}{SD}$ $Z = \frac{68 - 82}{8}$ $Z = \frac{-14}{8} = -1.75$ $PR = 4.01$

1. What was Bill's percentile rank in history and what percent of his classmates did he score better than?

To find the percentile rank that corresponds with each  $Z$  score we looked at Appendix A. Because both of the  $Z$  scores for Bill were negative, we looked at the column that told us the area beyond the  $Z$ . We found that Bill's history  $Z$  score of  $-2.00$  corresponds to a percentile rank of 2.27. Therefore, Bill scored better than 2.27% of the class in history.

2. What was Bill's percentile rank in political science and what percent of his classmates did he score better than?

We looked for the corresponding percentile rank for Bill's political science  $Z$  score. His  $Z$  score of  $-2.13$  corresponds to a percentile rank of 1.66, which means that Bill scored better than only 1.66% of the class in political science.

3. What was Ted's percentile rank in history and what percent of his classmates did he score better than?

Ted's history  $Z$  score of  $-2.30$  corresponds to a percentile rank of 1.07. Therefore, Ted scored better than only 1.07% of the history class.

4. What was Ted's percentile rank in political science and what percent of his classmates did he score better than?

Finally, Ted's political science  $Z$  score of  $-1.75$  corresponds to a percentile rank of 4.01, which means that he scored better than 4.01% of his political science classmates.

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Bill and Ted are doing horribly in both history and political science. It is no wonder that George Carlin took them back in time to learn history from the people who are making it and to experience political science in the making. We surely hope that after their “excellent adventure,” these Dudes would have scores higher than the class means.

When their history teacher computed the  $T$  scores, he used the formula that we gave you above:

$$T = 10(Z) + 50$$

5. What is Bill’s  $T$  score on his history exam?

**He converted Bill’s history  $Z$  score into a  $T$  score in the following way:**

$$T = 10(-2.00) + 50$$

$$T = -20 + 50$$

$$T = 30$$

**Bill’s history  $T$  score is 30.**

6. What is Ted’s  $T$  score on his history exam?

**Likewise, he converted Ted’s history  $Z$  score into a  $T$  score in the following way:**

$$T = 10(-2.30) + 50$$

$$T = -23 + 50$$

$$T = 27$$

**Ted’s history  $T$  score is 27.**

Bill and Ted are certainly examples that not everyone is average.

## Your New Best Friend—SPSS

If you know the mean and standard deviation for any data set, SPSS can quickly convert raw scores to standard scores. In the data-set window, you click on Transform and then Compute. If you are converting to  $Z$  scores, under Target Variable you might want to label your new variable  $z$  (something). If you are converting to  $T$  scores, you might want to call it  $t$  (something). Under Numeric Expression, you will use the appropriate standardization formula we gave you above. Maybe an example will make this clearer.

Let’s convert the midterm raw scores we introduced in Chapter 2 (Table 2.1) to  $Z$  scores. The midterm data set can be found on the Sage Web site ([www.sagepub.com/kurpius](http://www.sagepub.com/kurpius)). In the SPSS program, click on Transform and then Compute. In the new window, call your target variable  $z_{midterm}$ . Under Numeric Expression, put the formula “(midterm – 84)/9.41.” The

mean for the midterm grades was 84 and the standard deviation was 9.41. When you click OK, the computer will calculate a *zmidterm* score to match each raw midterm score. If you did this correctly, your *zmidterm* scores should be exactly the same as our *zmid* scores.

## Words of Encouragement



Unlike Bill and Ted, you have the potential to be curve-wreckers with your newfound knowledge of standard scores. If you have diligently followed everything we have done step-by-step, you are able to calculate and interpret standard scores. Good job!!!

