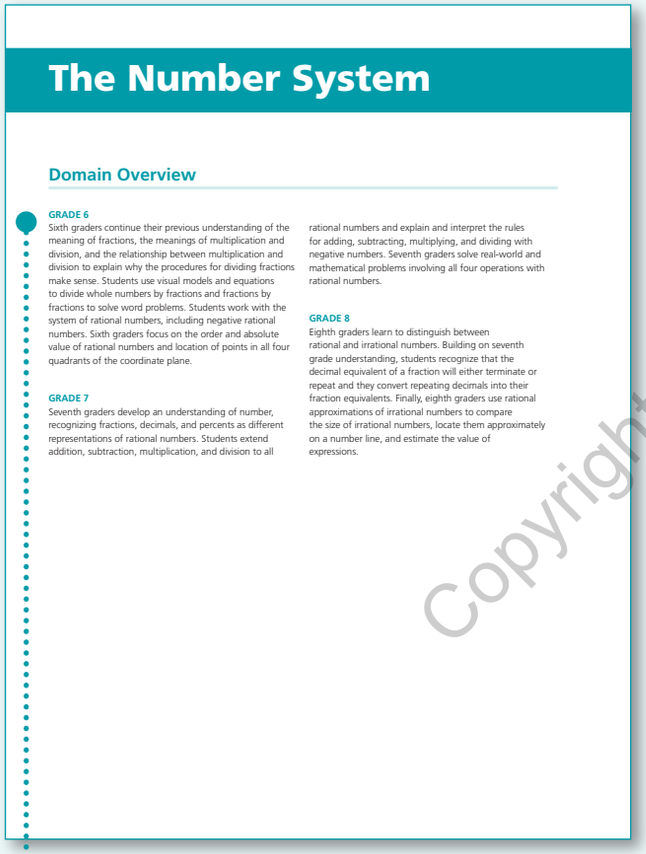
The Common Core Companion at a Glance



The Number System

Domain Overview

GRADE 6
Sixth graders continue their previous understanding of the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to explain why the procedures for dividing fractions make sense. Students use visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students work with the system of rational numbers, including negative rational numbers. Sixth graders focus on the order and absolute value of rational numbers and location of points in all four quadrants of the coordinate plane.

GRADE 7
Seventh graders develop an understanding of number, recognizing fractions, decimals, and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers and explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. Seventh graders solve real-world and mathematical problems involving all four operations with rational numbers.

GRADE 8
Eighth graders learn to distinguish between rational and irrational numbers. Building on seventh grade understanding, students recognize that the decimal equivalent of a fraction will either terminate or repeat and they convert repeating decimals into their fraction equivalents. Finally, eighth graders use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions.

Domain Overview: Gives a brief description of the big ideas, allowing you to see how the mathematical ideas develop across grade levels.

Suggested Materials for This Domain: Provides teachers with a list of materials that will be helpful in introducing the concepts in this domain.

SUGGESTED MATERIALS FOR THIS DOMAIN

6	7	8	
✓			Adding machine tape (optional, used to create number lines)
	✓		Algeblocks™ or Algebra Tiles™
✓			Coordinate grids
✓			Decimal blocks
✓			Factor trees
✓	✓	✓	Number lines
✓			Pattern blocks
✓	✓		Two-color counters

KEY VOCABULARY

6	7	8	
✓	✓		**absolute value** distance from 0 on a number line
	✓		**additive inverse** a number that, when added to another number, gives a sum of zero
✓			**algorithm** a set of steps used to solve a mathematical computation such as long division
✓			**common factor** a factor that two or more numbers have in common
	✓		**complex fraction** a fraction with a fraction in the numerator and/or a fraction in the denominator
✓			**coordinate plane** a plane formed by the intersection of a horizontal number line (called the *x*-axis) with a vertical number line called the *y*-axis. The number lines intersect at their zero points called the origin.
✓			**coordinates** set of numbers, or a single number, that locates a point on a line, on a plane, or in space
✓			**denominator** the bottom number of a fraction that shows how many equal parts the whole is divided into
✓			**distributive property** the property that states that multiplying a sum by a number is the same as multiplying each addend by the number and then adding the products. The distributive property is $a \times (b + c) = (a \times b) + (a \times c)$.
✓			**dividend** the number to be divided
✓			**divisor** the number that divides the dividend in a division problem

(Continued)

Key Vocabulary: Vocabulary included in the domain with grade levels indicated. This terminology can be used for building a word wall in the classroom. Students should be able to use these terms in talking about mathematics. Standard for Mathematical Practice 6: Attend to Precision calls for students to use mathematical terminology appropriately.

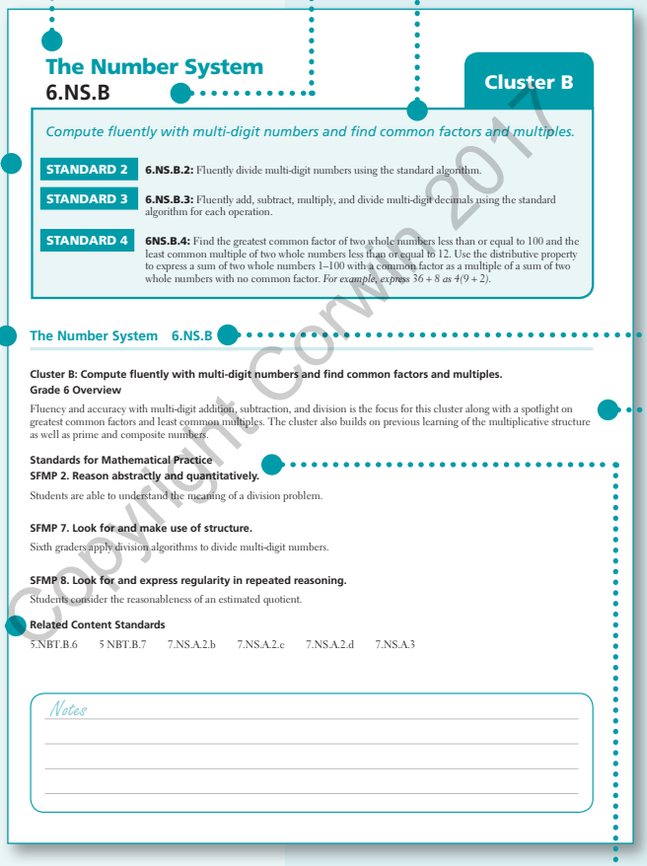
Domain: General mathematical topic for this group of standards.

Cluster: Statements that summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Identifying number for this cluster: Grade, domain, cluster

6 = Grade
NS = Domain
B = Cluster

Standards: Mathematical statements that define what students should understand and be able to do.



The Number System
6.NS.B

Cluster B

Compute fluently with multi-digit numbers and find common factors and multiples.

STANDARD 2 **6.NS.B.2:** Fluently divide multi-digit numbers using the standard algorithm.

STANDARD 3 **6.NS.B.3:** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

STANDARD 4 **6NS.B.4:** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express* 36 + 8 *as* 4(9 + 2).

Domain

The Number System 6.NS.B

Cluster B: Compute fluently with multi-digit numbers and find common factors and multiples.
Grade 6 Overview

Fluency and accuracy with multi-digit addition, subtraction, and division is the focus for this cluster along with a spotlight on greatest common factors and least common multiples. The cluster also builds on previous learning of the multiplicative structure as well as prime and composite numbers.

Standards for Mathematical Practice
SFMP 2. Reason abstractly and quantitatively.

Students are able to understand the meaning of a division problem.

SFMP 7. Look for and make use of structure.

Sixth graders apply division algorithms to divide multi-digit numbers.

SFMP 8. Look for and express regularity in repeated reasoning.

Students consider the reasonableness of an estimated quotient.

Related Content Standards

5.NBT.B.6 5 NBT.B.7 7.NS.A.2.b 7.NS.A.2.c 7.NS.A.2.d 7.NS.A.3

Notes

Each cluster begins with a brief description of the mathematics in that cluster.

Related Content Standards: Provides a list of standards connected to this topic in other grade levels as well as standards in this grade level related to this topic that are in other domains. Consider the related standards as you plan instruction for each cluster.

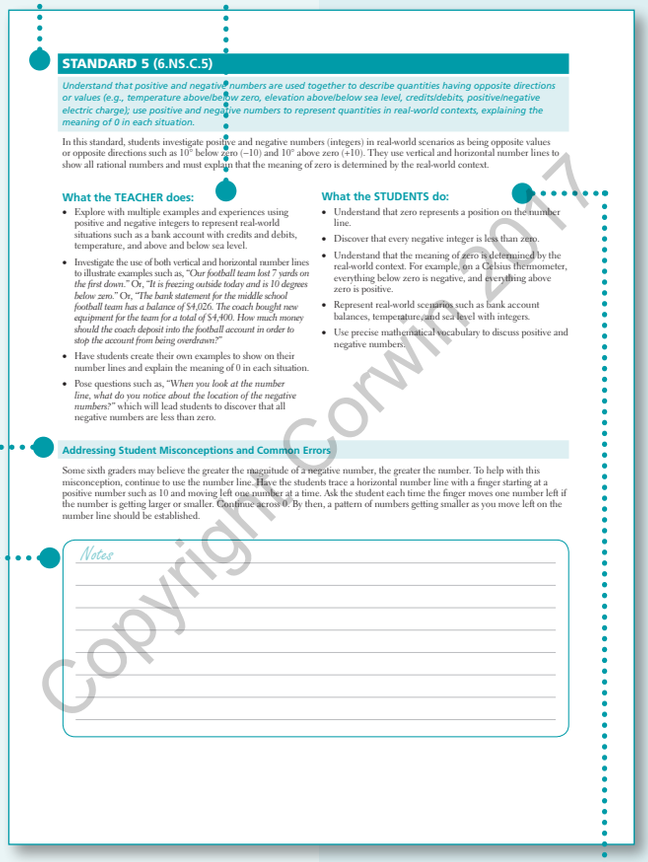
Standards for Mathematical Practice: Although it is likely you will use a variety of Standards for Mathematical Practice in teaching each cluster, this section gives examples of how you might incorporate some of the practices into your instruction on this topic.

You will find the following components for each standard in the cluster:

Standard: The standard as written in the Common Core, followed by an explanation of the meaning of the mathematics in that standard, including examples.

What the TEACHER does: An overview of actions the teacher might take in introducing and teaching the standard. This is not meant to be all-inclusive, but rather to give you an idea of what classroom instruction might look like. Illustrations may be included, detailing how to use materials to teach a concept when using models and representations called for in the standard.

Addressing Student Misconceptions and Common Errors: Each standard concludes with a misconception or common student error around the standard and suggested actions to address those misconceptions or errors.



STANDARD 5 (6.NS.C.5)

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

In this standard, students investigate positive and negative numbers (integers) in real-world scenarios as being opposite values or opposite directions such as 10° below zero (−10) and 10° above zero (+10). They use vertical and horizontal number lines to show all rational numbers and must explain that the meaning of zero is determined by the real-world context.

What the TEACHER does:

- Explore with multiple examples and experiences using positive and negative integers to represent real-world situations such as a bank account with credits and debits, temperature, and above and below sea level.
- Investigate the use of both vertical and horizontal number lines to illustrate examples such as, *"Our football team lost 7 yards on the first down."* Or, *"It is freezing outside today and is 10 degrees below zero."* Or, *"The bank statement for the middle school football team has a balance of $4,026. The coach bought new equipment for the team for a total of $4,400. How much money should the coach deposit into the football account in order to stop the account from being overdrawn?"*
- Have students create their own examples to show on their number lines and explain the meaning of 0 in each situation.
- Pose questions such as, *"When you look at the number line, what do you notice about the location of the negative numbers?"* which will lead students to discover that all negative numbers are less than zero.

What the STUDENTS do:

- Understand that zero represents a position on the number line.
- Discover that every negative integer is less than zero.
- Understand that the meaning of zero is determined by the real-world context. For example, on a Celsius thermometer, everything below zero is negative, and everything above zero is positive.
- Represent real-world scenarios such as bank account balances, temperature, and sea level with integers.
- Use precise mathematical vocabulary to discuss positive and negative numbers.

Addressing Student Misconceptions and Common Errors

Some sixth graders may believe the greater the magnitude of a negative number, the greater the number. To help with this misconception, continue to use the number line. Have the students trace a horizontal number line with a finger starting at a positive number such as 10 and moving left one number at a time. Ask the student each time the finger moves one number left if the number is getting larger or smaller. Continue across 0. By then, a pattern of numbers getting smaller as you move left on the number line should be established.

Notes

Notes: Included is blank space beneath each standard for taking notes while studying the mathematical content. This might include vocabulary, materials, resources you want to use, or an explanation of the standard in your own words.

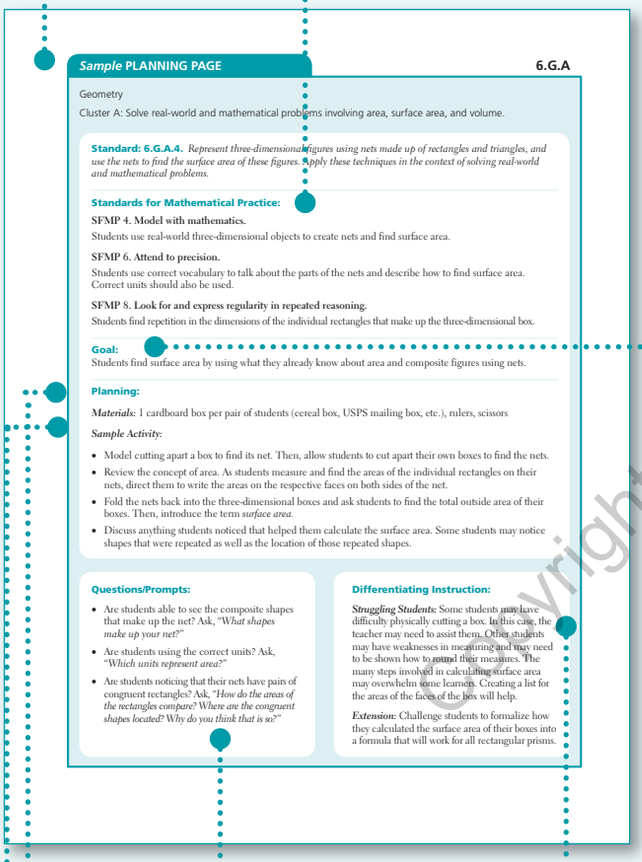
What the STUDENTS do: Some examples of what students may do as they explore and begin to understand the standard. This is not intended to be directive, but rather to frame what student actions may look like.

Sample Planning Page: Provided is a complete sample planning page for one standard at the end of each grade level. While these are not complete lesson plans, they provide ideas, activities, and a structure for planning.

Goal: The purpose of this activity and how it connects to previous and future ideas is stated.

Standards for Mathematical Practice: The Mathematical Practices emphasized in this sample plan are included.

Planning Page: A planning template is provided at the end of each grade level.



Sample PLANNING PAGE 6.G.A

Geometry

Cluster A: Solve real-world and mathematical problems involving area, surface area, and volume.

Standard: 6.G.A.4. *Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.*

Standards for Mathematical Practice:

SFMP 4. Model with mathematics.
Students use real-world three-dimensional objects to create nets and find surface area.

SFMP 6. Attend to precision.
Students use correct vocabulary to talk about the parts of the nets and describe how to find surface area. Correct units should also be used.

SFMP 8. Look for and express regularity in repeated reasoning.
Students find repetition in the dimensions of the individual rectangles that make up the three-dimensional box.

Goal:
Students find surface area by using what they already know about area and composite figures using nets.

Planning:

Materials: 1 cardboard box per pair of students (cereal box, USPS mailing box, etc.), rulers, scissors

Sample Activity:

- Model cutting apart a box to find its net. Then, allow students to cut apart their own boxes to find the nets.
- Review the concept of area. As students measure and find the areas of the individual rectangles on their nets, direct them to write the areas on the respective faces on both sides of the net.
- Fold the nets back into the three-dimensional boxes and ask students to find the total outside area of their boxes. Then, introduce the term *surface area.*
- Discuss anything students noticed that helped them calculate the surface area. Some students may notice shapes that were repeated as well as the location of those repeated shapes.

Questions/Prompts:

- Are students able to see the composite shapes that make up the net? Ask, *"What shapes make up your net?"*
- Are students using the correct units? Ask, *"Which units represent area?"*
- Are students noticing that their nets have pairs of congruent rectangles? Ask, *"How do the areas of the rectangles compare? Where are the congruent shapes located? Why do you think that is so?"*

Differentiating Instruction:

Struggling Students: Some students may have difficulty physically cutting a box. In this case, the teacher may need to assist them. Other students may have weaknesses in measuring and may need to be shown how to round their measures. The many steps involved in calculating surface area may overwhelm some learners. Creating a list for the areas of the faces of the box will help.

Extension: Challenge students to formalize how they calculated the surface area of their boxes into a formula that will work for all rectangular prisms.

PLANNING PAGE 6.G.A

Geometry

Cluster A: Solve real-world and mathematical problems involving area, surface area, and volume.

Standard:

Standards for Mathematical Practice:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Materials: The materials used in the Sample Activity are listed.

Questions/Prompts: This section provides questions or prompts you may use to help build student understanding and encourage student thinking.

Differentiating Instruction: Suggestions to address the need of struggling learners along with extension ideas to challenge other students are included here.

Sample Activity: An example of an activity that addresses this standard is provided.

Resources: In the Resources section at the end of the book you will find tables outlining the Standards for Mathematical Practice and Effective Teaching Practices from NCTM's *Principles to Actions*, the CCSS Where to Focus Mathematics, and reproducibles.

Table 1 Standards for Mathematical Practice

Standard for Mathematical Practice	What the Teacher Does	What the Students Do

1. Make sense of problems and persevere in solving them.	• Provide students with rich tasks and real-world problems that focus on and promote student understanding of an important mathematical concept.
	• Provide time for and facilitate the discussion of problem solutions.
	  ○ What are you asked to find?
	  ○ Have you solved a similar problem before?
	  ○ What is your plan for solving the problem?
	  ○ Can you explain how you solved the problem?
	  ○ Does your answer make sense?
	  ○ Did you use a different method to check your answer?
		• Actively engage in solving problems by working to understand the information that is in the problem and the question that is asked.
		• Use a variety of strategies that make sense to solve the problem.
		• Try a different strategy if the first strategy does not work.
		• Ask themselves if they used the most efficient way to solve the problem.
		• Ask themselves if their solution makes sense.
		• Solve real-world problems through the application of algebraic and geometric concepts.

2. Reason abstractly and quantitatively.	• Provide real-world scenarios to use real numbers and variables in mathematical expressions, equations, and inequalities. • Help students decontextualize to manipulate symbolic representations by applying properties of operations. • Help students understand the meaning of the number or variable as related to a problem.	• Use varied strategies, models, and drawings to think about the mathematics of a task and example. • Represent a wide variety of real-world situations through the use of real numbers and variables in mathematical expressions, equations, and inequalities. • Contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties. • Examine patterns in data and assess the degree of linearity of functions.

3. Construct viable arguments and critique the reasoning of others.	• Provide tasks that encourage students to construct mathematical arguments.
	• Expect students to explain their strategies and mathematical thinking to others.
	• Expect students to listen to the reasoning of others and respond to their thinking.
	• Help students to compare strategies and methods by asking questions such as:
	  ○ How can you prove that your answer is correct?
	  ○ What do you think about _____'s strategy?
	  ○ How is your method different from _____'s? How is it similar?
	  ○ Why is this true? Does it always work?
		• Explain orally or in writing their strategies and thinking using models, drawings, or symbolic representations.
		• Critique and evaluate their own thinking and the thinking of other students.
		• Ask questions to one another and to the teacher to clarify their understanding.
		• Look for similarities among different ways to solve problems.
		• Construct arguments using verbal or written explanations for expressions, equations, inequalities, models, and graphs, tables, and other data displays.

Table 2 Effective Teaching Practices

Teaching Practice	Purpose	What the Teacher Does	What the Students Do

1. Establish mathematics goals to focus learning.	• Set the stage to guide instructional decisions.
	• Expect students to understand the purpose of a lesson beyond simply repeating the words in the Standard.
		• Consider broad goals as well as the goals of the unit and the lesson, including:
		  ○ What is to be learned?
		  ○ Why is the goal important?
		  ○ Where do students need to go?
		  ○ How can learning be extended?
			• Make sense of the new concepts and skills, making connections to previously learned Grades 6–8 concepts.
			• Experience connections among the Standards and across domains.
			• Deepen their understanding and expect what they are learning makes sense.

2. Implement tasks that promote reasoning and problem solving.	• Provide opportunities for students to engage in exploration and make sense of important mathematics.
	• Encourage students to use procedures in ways that are connected to understanding.
		• Choose tasks that:
		  ○ are built on current student understandings,
		  ○ have various entry points with multiple ways for the problems to be solved,
		  ○ are interesting to students.
			• Work to make sense of the task and persevere in solving problems.
			• Use a variety of models and materials to make sense of the mathematics in the task.
			• Convince themselves and others the answer is reasonable.

3. Use and connect mathematical representations.	• Lead students to connect conceptual understanding of procedural skills using models and representations.	• Use tasks that allow students to use a variety of representations. • Encourage the use of different representations, including concrete manipulatives, models, and symbolic representations that support students in explaining their thinking and reasoning.	• Use materials to make sense of problem situations. • Connect representations to mathematical concepts and the structure of big ideas for ratios and proportional relationships, expressions, and equations, the number system, statistics, and probability, geometry, and functions.
4. Facilitate meaningful mathematical discourse.	• Provide students with opportunities to share ideas, clarify their understanding, and develop convincing arguments. • Allow discussion to advance mathematical thinking for the whole class.	• Engage students in explaining their mathematical reasoning in small group and classroom discussions. • Facilitate dialog among students that supports sense making of a variety of strategies and approaches. • Scaffold classroom discussions so that connections between representations and mathematical ideas occurs.	• Explain their ideas and reasoning in small groups and with the entire class. • Listen to the reasoning of others. • Ask questions of others to make sense of their ideas.

CCSS Where to Focus Grade 6 Mathematics

CCSS WHERE TO FOCUS GRADE 6 MATHEMATICS

MATH (MATHEMATICS) 6 (GRADE 6) F (FOCUS)

This document shows where students and teachers should spend the large majority of their time in order to meet the expectations of the Standards.

Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the Standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

Students should spend the large majority[1] of their time on the major work of the grade (■). Supporting work (□) and, where appropriate, additional work (○) can engage students in the major work of the grade.[2, 3]

MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 6

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: ■ Major Clusters □ Supporting Clusters ○ Additional Clusters

6.RP.A	■ Understand ratio concepts and use ratio reasoning to solve problems.
6.NS.A	■ Apply and extend previous understandings of multiplication and division to divide fractions by fraction.
6.NS.B	○ Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.C	■ Apply and extend previous understandings of numbers to the systems of rational numbers.
6.EE.A	■ Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.B	■ Reason about and solve one-variable equations and inequalities.
6.EE.C	■ Represent and analyze quantitative relationships between dependent and independent variables.
6.G.A	□ Solve real-world and mathematical problems involving area, surface area, and volume.
6.SP.A	○ Develop understanding of statistical variability.
6.SP.B	○ Summarize and describe distributions.

HIGHLIGHTS OF MAJOR WORK IN GRADES K–8

K–2	Addition and subtraction – concepts, skills, and problem solving; place value
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional relationships; early expressions and equations
7	Ratios and proportional relationships; arithmetic of rational numbers
8	Linear algebra and linear functions

REQUIRED FLUENCIES FOR GRADE 6

6.NS.B.2	Multi-digit division
6.NS.B.3	Multi-digit decimal operations

1 At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see Criterion #1 of the K–8 Publishers' Criteria for the Common Core State Standards for Mathematics www.achievethecore.org/publisherscriteria.

2 Refer also to criterion #3 in the K–8 Publishers' Criteria for the Common Core State Standards for Mathematics www.achievethecore.org/publisherscriteria.

3 Note, the critical areas are a survey of what will be taught at each grade level; the major work is the subset of topics that deserve the large majority of instructional time during a given year to best prepare students for college and careers.

STUDENT ACHIEVEMENT PARTNERS Find additional resources at achievethecore.org

Source: Created by Student Achievement Partners (SAP). http://achievethecore.org/content/upload/SAP_Focus_Math_6.pdf

CCSS Where to Focus Mathematics: The major content focus for each grade level is identified on the grade-level focus charts included in the Resources.

Reproducibles: A variety of reproducibles can be duplicated and used by students in the classroom when working with concrete materials.

Reproducible 1. Percent Wheel

Directions: Cut out two wheels on cardstock. Cut along the dotted line to the center of each wheel. Insert the wheels into each other through the cuts. Position the wheels so the lines face out. You should be able to see the lines on each side when the wheels are together.