4

MEASURES OF VARIABILITY

CHAPTER OUTLINE

Chapter 3 introduced descriptive statistics, the purpose of which is to numerically describe or summarize data for a variable. As we learned in Chapter 3, a set of data is often described in terms of the most common or frequently occurring score in the set of data by using a measure of central tendency such as the mode, median, or mean. Measures of central tendency numerically describe two aspects of a distribution of scores, modality and symmetry, based on what the scores have in common with each other. However, researchers also need to describe the amount of *differences* among the scores of a variable. This chapter introduces measures of variability, which are designed to represent the amount of differences in a distribution of data. In this chapter, we will present and evaluate different measures of variability in terms of their relative strengths and weaknesses. As in the previous chapters, our presentation will rely on the findings and analysis from a published research study.

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4.1 AN EXAMPLE FROM THE RESEARCH: HOW MANY "SOMETIMES" IN AN "ALWAYS"?

Throughout our daily lives, people are constantly asked to fill questionnaires and surveys. Restaurants ask customers to evaluate the quality of the food and service they provide; websites want to know how satisfied shoppers are with their online purchases; political organizations are interested in learning what voters think about their elected officials. In responding to these types of scenarios, people are often asked to describe their feelings or beliefs using words such as *excellent, good,* or *poor.* But do words such as these have the same meaning for different people? If research participants disagree about the meaning of key terms and phrases, it's difficult to combine or compare their responses.

Two researchers, Suzanne Skevington and Christine Tucker, conducted a study assessing differences in interpretation of labels for questionnaire items measuring the frequency of health-related behaviors, such as, "How often do you suffer pain?" (Skevington & Tucker, 1999). As a part of their study, they examined how British people define frequency-related terms such as *seldom, usually,* and *rarely.* More specifically, they assigned numeric values to participants' evaluation of these words in order to measure the degree to which people differ in their interpretation of these words.

In their study, Skevington and Tucker (1999) presented 20 British adults a variety of frequency-related words. For each word, they included a line 100 mm (about 4 inches) in length. At the two ends of the line were the labels *Never* and *Always,* representing the lowest (0%) and highest (100%) possible values for frequency. After reading each word, participants were asked to place an X on the point in the line that they felt best represented the frequency of the word, relative to the end points of Never and Always. An example of their methodology is provided in Figure 4.1.

After the participants completed their responses, the researchers used a ruler to measure the distance from the word *Never* to the "X" provided by the participant. This distance, which was measured in millimeters, represented the perceived "frequency" of the word. For example, in Figure 4.1, the X for the word *sometimes* is in the exact middle of the line; consequently, this word is given a frequency rating of 50% because it is 50 mm from the left end. In this study, the variable, which we will call "frequency rating," had possible values ranging from 0 to 100.

In this example, we will focus on the study's findings regarding the word *sometimes.* The frequency ratings for the word *sometimes* for the 20 participants are listed in Table 4.1(a). Table 4.1(b) organizes the raw data into a grouped frequency distribution table, and Figure 4.2 illustrates the distribution using a frequency polygon. A frequency polygon is used rather than a histogram or pie chart because the variable "frequency rating" is measured at the ratio level of measurement, with the value of 0 representing the complete absence of distance from the left end of the 100-mm line.

As discussed in Chapter 3, the data for a variable can be summarized using a measure of central tendency. Using Formula 3-3 from Chapter 3, the mean frequency rating of the word *sometimes* for the 20 participants is calculated as follows:

$$\begin{aligned}\bar{X} &= \frac{\Sigma X}{N} \\ &= \frac{46+15+21+\ldots+22+66+26}{20} = \frac{666}{20} \\ &= 33.30\end{aligned}$$

From this calculation, we conclude that the participants in the study believed, on average, the word *sometimes* represents a 33.30% frequency. However, the grouped frequency distribution table in Table 4.1(b) reveals that

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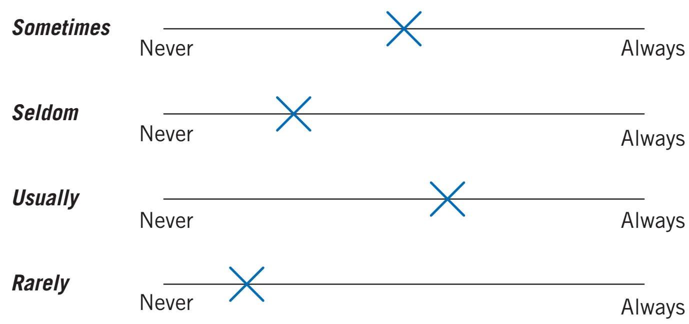
FIGURE 4.1 ● EXAMPLE OF RATINGS OF FREQUENCY-RELATED WORDS ALONG A 100-MM LINE



participants provided a wide range of ratings, some of which were far from the average rating of 33.30%. For example, the ratings provided by 11 of the 20 participants were lower than 30%, while 6 of the 20 participants provided ratings of 40% or higher. In other words, the people in this sample greatly varied in their interpretation of the word *sometimes.*

Given the variety of ratings these participants gave the word *sometimes,* in order to accurately describe the distribution of responses for this variable, one must not only include a measure of central tendency (which focuses on what the scores have in common with each other) but also represent the degree to which the scores differ, or vary. The next section introduces the concept of variability.

4.2 UNDERSTANDING VARIABILITY

We begin our discussion of variability by examining the three distributions presented in Figure 4.3. Chapter 2 discussed three aspects of distributions: modality, symmetry, and variability. The modality and symmetry of a distribution can be described using measures of central tendency such as those introduced in Chapter 3. However, because all three distributions in Figure 4.3 are unimodal and symmetric, they would be described as having the same mode, median, and mean. It is obvious, however, that the three distributions are very different. As the remainder of this chapter will illustrate, these differences can be portrayed using a statistic that describes the third aspect of distributions: variability.

The word *variability* typically evokes words such as *differences* or *changes.* In a statistical sense, variability refers to the amount of spread or scatter of scores in a distribution. The concept of variability is a critical issue in the behavioral sciences, where research frequently examines differences in such things as characteristics, attitudes, and cognitive abilities. Different people, for example, express different levels of extroversion, different attitudes toward capital punishment, and different learning styles. Ultimately, the primary goal of a science such as psychology is to describe, understand, explain, and predict variability.

Researchers have developed statistics designed to measure variability. A **measure of variability** is a descriptive statistic of the amount of differences in a set of data for a variable. The purpose of measures of variability is to numerically represent a set of data based on how the scores differ or vary from each other. Similar to measures of central tendency, there are multiple measures of variability. The next part of this chapter presents and discusses four measures of variability:

- the range,
- the interquartile range,

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TABLE 4.1 ⬢ FREQUENCY RATING OF THE WORD SOMETIMES BY 20 PARTICIPANTS

(a) Raw Data

Participant	Frequency Rating	Participant	Frequency Rating	Participant	Frequency Rating
1	**46**	8	**27**	15	**49**
2	**15**	9	**20**	16	**32**
3	**21**	10	**23**	17	**45**
4	**49**	11	**58**	18	**22**
5	**23**	12	**24**	19	**66**
6	**39**	13	**36**	20	**26**
7	**25**	14	**20**		

(b) Grouped Frequency Distribution Table

Frequency Rating	*f*	%
70–100	0	0%
60–69	1	5%
50–59	1	5%
40–49	4	20%
30–39	3	15%
20–29	10	50%
10–19	1	5%
0–9	0	0%
Total	20	100%

- the variance, and
- the standard deviation.

Each of these measures of variability will be defined, illustrated in terms of their necessary calculations, and evaluated based on their relative strengths and weaknesses.

4.3 THE RANGE

One way to describe the amount of variability in a distribution of data for a variable is to focus on the two ends of the distribution. Therefore, the first measure of variability we will discuss is the **range**, defined as the mathematical difference between the lowest and highest scores in a set of data:

$$\text{Range} = \text{highest score} - \text{lowest score} \tag{4-1}$$

The range is computed by identifying the lowest and highest scores in a set of data and then subtracting the lowest score from the highest score to compute the difference between the two scores.

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FIGURE 4.2 ⬢ FREQUENCY POLYGON, FREQUENCY RATINGS OF *SOMETIMES*

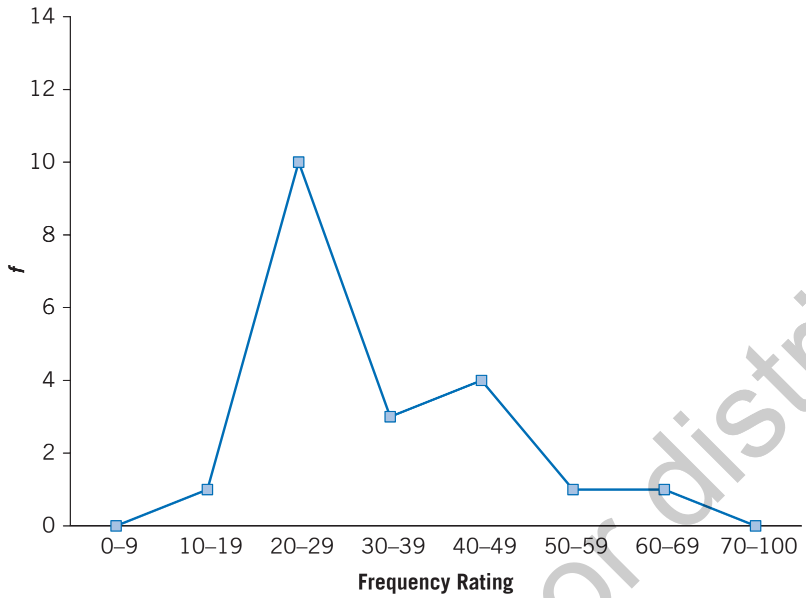
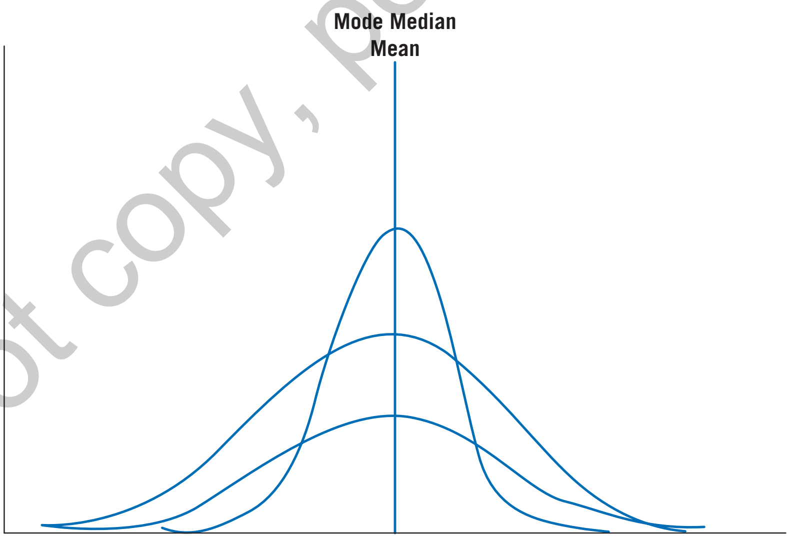


FIGURE 4.3 ⬢ THREE DISTRIBUTIONS WITH THE SAME MODALITY AND SYMMETRY BUT DIFFERENT VARIABILITY



To illustrate how to calculate the range, let's return to the small set of data introduced in Chapter 3 to discuss measures of central tendency: 2, 3, 1, 4, 3, 1, 4, 6, and 3. Among the nine scores in this set of data, the lowest score is 1 and the highest score is 6. Using Formula 4-1, the range for this set of data is the following:

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$$\text{Range} = \text{highest score} - \text{lowest score}$$
$$= 6 - 1$$
$$= 5$$

As a second example, to calculate the range for the frequency rating variable in Table 4.1(a), the ratings of the 20 participants are sorted from lowest to highest:

Lowest	*2nd*	*3rd*	*4th*	*5th*	*6th*	*7th*	*8th*	*9th*	*10th*	*11th*	*12th*	*13th*	*14th*	*15th*	*16th*	*17th*	*18th*	*19th*	*Highest*
15	**20**	**20**	**21**	**22**	**23**	**23**	**24**	**25**	**26**	**27**	**32**	**36**	**39**	**45**	**46**	**49**	**49**	**58**	**66**

Among the 20 participants, the lowest and highest ratings are 15 and 66, respectively. Therefore, the range for the frequency rating variable is

$$\text{Range} = \text{highest score} - \text{lowest score}$$
$$= 66 - 15$$
$$= 51$$

When researchers report the range for a variable, they generally provide the actual values of the lowest and highest scores. For the frequency rating variable, for example, the range might be reported as the following: "In this sample of 20 participants, frequency ratings for the word *sometimes* had a range of 51% (low = 15%, high = 66%)." Including the lowest and highest scores may provide information about the sample from which the data were collected. For example, even though the range of incomes ($35,000) may be the same in two samples, a sample in which the lowest and highest incomes are $5,000 and $40,000 would be considered differently from a sample in which the lowest and highest incomes are $175,000 and $210,000.

Strengths and Weaknesses of the Range

As a measure of variability, the range has comparative strengths and weaknesses. The primary strength of the range is that it is easy and quick to compute, particularly if the sample is small or if computer software is used to sort the scores from lowest to highest. A second strength of the range, as mentioned earlier, is that indicating the lowest and highest scores for a variable provides information about the sample from which the data were collected.

However, because the range is calculated from only two scores in the distribution (the lowest and highest), it may not accurately reflect the amount of variability in the entire distribution of scores. Consider, for example, the following two sets of scores sorted from lowest to highest:

Set 1: ***1*** 2 2 3 3 3 4 4 ***5***

Set 2: ***1*** 3 4 5 5 5 5 5 ***5***

Although the range for both sets of data is the same (5 − 1 = 4), there is much more variability in the first set than in the second. The data in Set 2 illustrate another weakness of the range: It is affected by outliers (in this case, the one score of 1). Because the range does not take into account all of the scores in the distribution, a fundamental weakness of the range is that it cannot be used in statistical analyses designed to test hypotheses about distributions.

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4.4 THE INTERQUARTILE RANGE

One of the limitations of the range as a measure of variability is that it is affected by extreme scores known as outliers. One way to overcome the influence of outliers on the range is to calculate the **interquartile range,** which is the range of the middle 50% of the scores in a set of data. The interquartile range is calculated using Formula 4-2:

$$\text{Interquartile range} = (N - \frac{N}{4})\text{th score} - (\frac{N}{4} + 1)\text{th score} \qquad (4\text{-}2)$$

where N is the total number of scores. The interquartile range is calculated by removing the highest and lowest 25% of the distribution and then calculating the range of the remaining scores. The primary purpose of the interquartile range is to decrease the influence of outliers in representing the variability in a set of data.

As a simple example, consider the following eight scores ($N = 8$), sorted from lowest to highest:

Lowest	*2nd*	*3rd*	*4th*	*5th*	*6th*	*7th*	*Highest*
11	**15**	**19**	**24**	**31**	**39**	**42**	**89**

Using Formula 4-2, the first step is to identify the $(N - \frac{N}{4})$ and the $(\frac{N}{4} + 1)$ th scores. Because $N = 8$ in this example,

$$\begin{aligned}\text{Interquartile range} &= (N - \frac{N}{4})\text{th score} - (\frac{N}{4} + 1)\text{th score}\\ &= (8 - \frac{8}{4})\text{th score} - (\frac{8}{4} + 1)\text{th score}\\ &= (8 - 2)\text{th score} - (2 + 1)\text{th score}\\ &= 6\text{th score} - 3\text{rd score} = 39 - 19\\ &= 20\end{aligned}$$

So, for this small set of data, the interquartile range is the difference between the values of the sixth and the third scores. Among the sorted scores, the sixth score is 39 and the third score is 19; therefore, the interquartile range is equal to (39 – 19), or 20. Note that the interquartile range of 20 is much smaller than the range of this set of data, which is (89 – 11), or 78.

To calculate the interquartile range for the frequency rating variable, because $N = 20$ in this example, we start by entering the value 20 into Formula 4-2:

$$\begin{aligned}\text{Interquartile range} &= (N - \frac{N}{4})\text{th score} - (\frac{N}{4} + 1)\text{th score}\\ &= (20 - \frac{20}{4})\text{th score} - (\frac{20}{4} + 1)\text{th score}\\ &= (20 - 5)\text{th score} - (5 + 1)\text{th score}\\ &= 15\text{th score} - 6\text{th score} = 45 - 23\\ &= 22\end{aligned}$$

To calculate the interquartile range for the frequency rating variable, the 15th and 6th scores must be identified. Earlier, the 20 ratings were sorted from lowest to highest to calculate the range; looking at the sorted ratings, we find the 15th score is equal to 45 and the 6th score is equal to 23. Therefore, the interquartile range for this set of data is equal to (45 – 23), or 22.

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Strengths and Weaknesses of the Interquartile Range

Compared with the range, the primary purpose of the interquartile range is to represent the variability in a set of data while lessening the influence of outliers. However, using the interquartile range has the potential to misrepresent a set of data by ignoring half (the top and bottom 25%) of the scores. It is somewhat counterintuitive to measure the variability in a set of data with only half of the data. Also, because the interquartile range, like the range, does not take into account all of the scores in the distribution, it cannot be used in statistical analyses designed to test hypotheses about distributions.

Given its limitations, under what conditions is the interquartile range most appropriately used? Concern about the impact of outliers was discussed in Chapter 3, where one advantage of the median as a measure of central tendency is that it is not affected by outliers. Therefore, the interquartile range is typically reported along with the median to represent the variability and central tendency in distributions that are skewed or have outliers.

LEARNING CHECK 1:

Reviewing What You've Learned So Far

1. Review questions
   a. Why is it important to calculate measures of variability for a variable in addition to measures of central tendency?
   b. What are the relative advantages and disadvantages of the range as a measure of variability?
   c. What is the difference between the range and the interquartile range?
   d. For what types of distributions might you report the interquartile range?
2. Calculate the range for each of the following sets of data:
   a. 2, 1, 4, 2, 7, 3
   b. 15, 6, 17, 9, 12, 5, 16, 7
   c. 21, 14, 13, 17, 30, 17, 14, 11, 13, 14, 19
   d. 3.0, 2.8, 3.2, 3.6, 3.7, 3.2, 3.5, 3.6, 3.1, 3.2, 3.0, 3.2, 3.8, 3.3
3. Calculate the interquartile range for each of the following sets of data:
   a. 9, 3, 2, 7, 15, 10, 14, 8
   b. 15, 6, 17, 9, 12, 5, 16, 7
   c. 13, 9, 9, 16, 10, 9, 5, 7, 9, 8, 10, 9
   d. 3.0, 2.8, 3.2, 3.6, 3.7, 3.2, 3.5, 3.2, 3.0, 3.2, 3.8, 3.3

4.5 THE VARIANCE (s^2)

The range and interquartile range describe the variability of a distribution of data based on two scores at or near the ends of the distribution. However, other measures of variability are based on all of the scores in a set of data. These other measures are an essential component of statistics designed to test research hypotheses.

To illustrate measures of variability based on all of the scores in a set of data, let's return to the simple data set of nine scores used earlier to illustrate the range: 2, 3, 1, 4, 3, 1, 4, 6, and 3. The mean ($\bar{X}$) of this sample of nine scores is calculated below:

$$\begin{aligned}\bar{X} &= \frac{\Sigma X}{N} \\ &= \frac{2+3+1+4+3+1+4+6+3}{9} = \frac{27}{9} \\ &= 3.00\end{aligned}$$

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One way to measure the variability in a set of data is based on the extent to which each score differs from the mean. You could, for example, ask the question, "On average, how different is each score from the mean?" In the set of nine scores, variability could be represented by the average difference between each score (X) and the mean of 3.00 ($\overline{X} = 3.00$). Referring to this difference as a "deviation," the third column in Table 4.2 calculates the deviation of each of the nine scores from the mean, symbolized by $(X - \overline{X})$.

Calculating the average of the deviations consists of dividing the summed deviations by the number of deviations, which is equal to N. The formula for the average deviation from the mean is provided below:

$$\text{Average deviation from the mean} = \frac{\Sigma(X - \overline{X})}{N}$$

The numerator of this formula is "the sum of deviations from the mean." Note that parentheses are placed around $X - \overline{X}$ to indicate that this deviation should be calculated for each score *before* adding the deviations together; if we did not include the parentheses, the notation $\Sigma X - \overline{X}$ would imply that the first step is to calculate the sum of the scores (ΣX) and then subtract the mean ($\overline{X}$) from this sum. As in all mathematical calculations, the placement of parentheses indicates the order in which mathematical operations are performed.

Using the deviations calculated in Table 4.2, the average deviation from the mean for the set of nine scores is calculated below:

$$\begin{aligned}\text{Average deviation from the mean} &= \frac{\Sigma(X - \overline{X})}{N} \\ &= \frac{-1.00 + .00 + -2.00 + \ldots + 1.00 + 3.00 + .00}{9} = \frac{0}{9} \\ &= .00\end{aligned}$$

Here, the average deviation from the mean is equal to zero (0). From this, we could conclude, "There is zero variability among the nine scores." However, concluding there is "zero" variability implies that all nine scores are exactly the same, which we know is not true. How did we reach the erroneous conclusion that all the scores are identical?

In *any* set of data, the average deviation from the mean is *always* equal to zero. This is because, as we explained in Chapter 3, the sum of the positive deviations from the mean is always equal to the sum of the negative deviations, which results in the sum of the deviations being equal to zero. Because the sum of the deviations is always equal to zero, the average deviation from the mean is also always equal to zero, regardless of the actual amount of variability among the scores.

Definitional Formula for the Variance

As we've just discussed, calculating a measure of variability that is based on the deviations from the mean is complicated by the fact that the sum of the negative and positive deviations will always be equal to each other. How can we overcome this issue? One way to resolve the "balancing act" of negative and positive deviations is to eliminate the negative deviations by squaring each deviation: $(X - \overline{X})^2$. The square of any number, negative or positive, is always a positive number.

So, rather than calculating the average deviation from the mean, the amount of variability in a sample of data may be measured by calculating the **variance (s^2)**, defined as the average squared deviation from the mean. Formula 4-3 provides what is known as the definitional formula for the variance:

$$s^2 = \frac{\Sigma(X - \overline{X})^2}{N - 1} \tag{4-3}$$

where X is a score for the variable, $\overline{X}$ is the mean of the sample, and N is the total number of scores.

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Two important aspects of the formula for the variance should be considered here. First, in the numerator, parentheses are placed around the deviation between each score and the mean $(X - \bar{X})$ to first calculate the deviation before performing any squaring or summing. Second, in the denominator, the sum of the squared deviations is not divided by the total number of scores (N) but instead is divided by the total number of scores minus 1 ($N - 1$). The rationale for dividing by $N - 1$ rather than by N will be discussed after we illustrate the calculations for the variance.

Table 4.3 begins the calculation of the variance for the set of nine scores. The first three columns of this table are identical to the same columns in Table 4.2—the fourth column squares each of the deviations. For example, the squared deviation for the first score is $(-1.00)^2$ or 1.00. Note that all of the squared deviations are positive numbers.

Using the squared deviations calculated in Table 4.3, the variance for the set of nine scores can be calculated using Formula 4-3:

$$\begin{aligned} s^2 &= \frac{\Sigma(X - \bar{X})^2}{N - 1} \\ &= \frac{1.00 + .00 + 4.00 + \ldots + 1.00 + 9.00 + .00}{9 - 1} = \frac{20.00}{8} \\ &= 2.50 \end{aligned}$$

From this calculation, we may conclude that the variance in this set of data is equal to 2.50. This means that, for the sample of nine scores, the average squared deviation of a score from the mean is 2.50.

In calculating the variance, because a squared deviation is always a positive number, the sum of the squared deviations $(\Sigma(X - \bar{X})^2)$ and subsequently the variance (s^2) must also always be positive numbers. This is an important point that helps students identify calculation errors in homework and paper assignments: *Obtaining a negative value for the sum of squared deviations or the variance means you have made a mistake in your calculations.*

As a second example, let's calculate the variance for the frequency rating variable. Using the sample mean $(\bar{X})$ of 33.30 calculated earlier in this chapter, the last column in Table 4.4 shows the squared deviation for each of the 20 ratings in this sample.

TABLE 4.2 ● CALCULATION OF THE DEVIATION OF EACH SCORE FROM THE MEAN $(X - \bar{X})$, SIMPLE EXAMPLE

Score (X)	Mean ($\bar{X}$)	Score – Mean ($X - \bar{X}$)
2	3.00	–1.00
3	3.00	.00
1	3.00	–2.00
4	3.00	1.00
3	3.00	.00
1	3.00	–2.00
4	3.00	1.00
6	3.00	3.00
3	3.00	.00

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TABLE 4.3 ● CALCULATION OF SQUARED DEVIATION FROM THE MEAN ($(X - \bar{X})^2$), SIMPLE EXAMPLE

Score (X)	Mean ($\bar{X}$)	Score – Mean ($X - \bar{X}$)	(Score – Mean)^2 ($(X - \bar{X})^2$)
2	3.00	–1.00	1.00
3	3.00	.00	.00
1	3.00	–2.00	4.00
4	3.00	1.00	1.00
3	3.00	.00	.00
1	3.00	–2.00	4.00
4	3.00	1.00	1.00
6	3.00	3.00	9.00
3	3.00	.00	.00

Using the squared deviations, the variance for the frequency rating variable is calculated using Formula 4-3 as follows:

$$s^2 = \frac{\Sigma(X - \bar{X})^2}{N - 1}$$
$$= \frac{161.29 + 334.89 + 151.29 + \ldots + 127.69 + 1{,}069.29 + 53.29}{20 - 1} = \frac{3{,}940.20}{19}$$
$$= 207.38$$

For this sample of 20 participants, the variance, which is to say the average squared deviation of a frequency rating from the mean of 33.30, is equal to 207.38.

Computational Formula for the Variance

The formula for the variance presented in Formula 4-3 represents the literal definition of variance: the average squared deviation from the mean. For this reason, it is referred to as a **definitional formula,** which is a formula based on the actual or literal definition of a concept. However, using a definitional formula to analyze a set of data can be tedious (because it requires calculating the deviation of each score from the mean) and complicated (because the mean often possesses decimal places [e.g., 33.30]). Because using the definitional formula for the variance in a large or complicated data set increases the chances of making computational errors, the formula can be algebraically manipulated to create what is known as a **computational formula,** defined as a formula not based on the definition of a concept but is designed to simplify mathematical calculations.

Formula 4-4 provides the computational formula for the variance:

$$s^2 = \frac{\Sigma X^2 - \frac{(\Sigma X^2)}{N}}{N - 1} \qquad (4\text{-}4)$$

where ΣX^2 is the sum of squared scores, $(\Sigma X)^2$ is the sum of scores squared, and N is the total number of scores. It is critical to understand that ΣX^2 and $(\Sigma X)^2$ involve a different order of operations. ΣX^2, which is the sum of squared scores, involves first squaring each score and then summing the squared scores (i.e., first squaring, then summing). On the other hand, $(\Sigma X)^2$, which is the sum of scores squared, involves first summing a set of scores and then squaring this sum (i.e., first summing, then squaring).

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TABLE 4.4 ⬢ CALCULATION OF SQUARED DEVIATION FROM THE MEAN ($(X - \bar{X})^2$), FREQUENCY RATING VARIABLE

Frequency Rating (X)	Mean ($\bar{X}$)	Frequency Rating – Mean ($X - \bar{X}$)	(Frequency Rating – Mean)^2 ($(X - \bar{X})^2$)
46	33.30	12.70	161.29
15	33.30	–18.30	334.89
21	33.30	–12.30	151.29
49	33.30	15.70	246.49
23	33.30	–10.30	106.09
39	33.30	5.70	32.49
25	33.30	–8.30	68.89
27	33.30	–6.30	39.69
20	33.30	–13.30	176.89
23	33.30	–10.30	106.09
58	33.30	24.70	610.09
24	33.30	–9.30	86.49
36	33.30	2.70	7.29
20	33.30	–13.30	176.89
49	33.30	15.70	246.49
32	33.30	–1.30	1.69
45	33.30	11.70	136.89
22	33.30	–11.30	127.69
66	33.30	32.70	1069.29
26	33.30	–7.30	53.29

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Table 4.5 begins the process of calculating the variance for the set of nine scores using the computational formula. The bottom of the first column contains the sum of the nine scores ($\Sigma X = 27$); the sum of scores squared ($(\Sigma X)^2$) is equal to $(27)^2$ or 729. The second column of Table 4.5 provides the squared values of each of the scores—the sum of squared scores (ΣX^2) is located at the bottom of this column ($\Sigma X^2 = 101$). The results of these calculations are then inserted into Formula 4-4 as follows:

$$s^2 = \frac{\Sigma X^2 - \frac{(\Sigma X^2)}{N}}{N-1}$$

$$= \frac{101 - \frac{729}{9}}{9-1} = \frac{101 - 81.00}{8} = \frac{20.00}{8}$$

$$= 2.50$$

It is important to note that the same value for the variance ($s^2 = 2.50$) was obtained whether we use the computational formula in Formula 4-4 or the definitional formula in Formula 4-3.

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TABLE 4.5 ⬢ CALCULATING THE SUM OF SQUARED SCORES (ΣX^2) AND SUM OF SCORES SQUARED ($(\Sigma X)^2$), SIMPLE EXAMPLE

Score (X)	(Score)^2 (X^2)
2	4
3	9
1	1
4	16
3	9
1	1
4	16
6	36
3	9
$\Sigma X = 27$	$\Sigma X^2 = 101$
$(\Sigma X)^2 = 729$	

To calculate the variance for the frequency rating variable using the computational formula, Table 4.6 calculates the sum of squared scores (ΣX^2) and the sum of scores squared ($(\Sigma X)^2$) for the 20 participants. Once these two quantities have been calculated, the variance may be calculated as follows:

$$s^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N-1}$$

$$= \frac{26{,}118 - \frac{443{,}556}{20}}{20-1} = \frac{26{,}118 - 22{,}177.80}{19} = \frac{3{,}940.20}{19}$$

$$= 207.38$$

The value of 207.38 for the variance is again identical using the definitional or the computational formula. One concern with using the computational formula is that the formula can confuse students because it does not appear to represent the concept of variance, which is based on the difference between each score and the mean $(X - \bar{X})$. However, we've included the computational formula because it simplifies the calculation of the variance, thereby reducing the chance of making computational errors.

Why Not Use the Absolute Value of the Deviation in Calculating the Variance?

To compute the variance using the definitional formula provided in Formula 4-3, the deviation between each score and the mean must be squared to eliminate negative deviations. Instead of doing all of this squaring, you may be asking yourself, "Wouldn't it be easier to eliminate the negative deviations simply by using the absolute value of each deviation? If so, you could simply calculate the average of the absolute values." The absolute value of a number, symbolized by parallel vertical lines, ignores the sign (+/–) of the number. For example, $|2 - 3.00|$, which is the absolute value of the deviation 2 – 3.00, is equal to 1.00.

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TABLE 4.6 ⬢ SUM OF SQUARED SCORES (ΣX^2) AND SUM OF SCORES SQUARED ($(\Sigma X)^2$), FREQUENCY RATING VARIABLE

Frequency Rating (X)	(Frequency Rating)^2 (X^2)	Frequency Rating (X)	(Frequency Rating)^2 (X^2)
46	2,116	58	3,364
15	225	24	576
21	441	36	1,296
49	2,401	20	400
23	529	49	2,401
39	1,521	32	1,024
25	625	45	2,025
27	729	22	484
20	400	66	4,356
23	529	26	676
		$\Sigma X = 666$	$\Sigma X^2 = 26{,}118$
		$(\Sigma X)^2 = 443{,}556$	

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The logic behind using absolute values of the deviations may be intuitively appealing. However, the statistical procedures discussed in this book that involve variability require algebraic manipulations that cannot be carried out using absolute values. For this reason, it is necessary to use formulas for the variance that are based on the squaring of the deviations.

Why Divide by $N - 1$ Rather Than N in Calculating the Variance?

In calculating the variance, students often wonder why the numerator is divided by $N - 1$, rather than by N. Given that the variance is the *average* squared deviation, it would seemingly make sense to divide the sum of the squared deviations by N. One reason for dividing by $N - 1$ is to estimate the variability in a population using data collected from a sample of the population. To illustrate this concept, consider an example in which data are collected from a sample of 100 freshmen at a local university in order to represent the entire population of freshmen nationwide. It is reasonable to suspect that the smaller sample of local university students would be less diverse than the entire population. That is, the sample may not accurately represent all of the possible values for ethnicity, economic background, age, attitudes, and so on that exist in the entire population. Consequently, the amount of variability in the sample will be less than what is believed to exist in the population.

Because the amount of variability in a sample is less than the variability in the population from which the sample is drawn, the sample variance underestimates the variance in the population. As a result, the sample variance is a **biased estimate** of the population variance; a biased estimate is a statistic for a sample that systematically underestimates or overestimates the population from which the sample was drawn. What is needed, therefore, is to correct the sample variance to make it an **unbiased estimate** of the population variance, which is a statistic based on a sample that is equally likely to underestimate or overestimate the population from which the sample was drawn.

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Given that the sample variance systematically underestimates the population variance, we need to increase the value of the sample variance to more accurately estimate the variability in the population. This sample variance is increased by dividing the numerator, the sum of squared deviations, by $N - 1$ rather than by N; dividing by a smaller number makes the result larger.

4.6 THE STANDARD DEVIATION (s)

The variance is the average squared deviation of a score from the mean. However, researchers typically want to represent the variability in a set of data using the average deviation, not the average *squared* deviation. This representation is accomplished by calculating a measure of variability known as the **standard deviation (s)**, defined as the square root of the variance. Mathematically, the standard deviation is the square root of the average squared deviation from the mean, and it represents the average deviation of a score from the mean.

LEARNING CHECK 2:

Reviewing What You've Learned So Far

1. Review questions
    a. Why does calculating the variance involve squaring the deviation of each score from the mean?
    b. What is the purpose of computational formulas?
    c. In calculating a measure of variability, why can't you use the absolute value of the deviation of each score from the mean rather than the squared deviation?
    d. In calculating the variance, why is the sum of squared deviations divided by $N - 1$ rather than N?
    e. What is the difference between a biased estimate and an unbiased estimate?
2. Calculate the variance (s^2) using the definitional and computational formulas for each of the following data sets.
    a. 2, 3, 3, 5, 7
    b. 5, 4, 7, 5, 10, 5, 6
    c. 10, 13, 13, 14, 15, 16, 17, 24
    d. 73, 66, 91, 84, 69, 87, 62, 79, 82, 90
    e. 11, 9, 9, 12, 10, 9, 7, 8, 9, 8, 10, 9

Definitional Formula for the Standard Deviation

The standard deviation, represented by the symbol s, is calculated by computing the square root of the variance. The purpose of calculating the square root is to "undo" the effect of squaring the deviations. Formula 4-5 provides the definitional formula for the standard deviation—this formula places the definitional formula for the variance (Formula 4-3) under a square root symbol:

$$s = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N - 1}} \qquad (4\text{-}5)$$

For the simple data set of nine scores used throughout this chapter, using the squared deviations calculated in Table 4.3, the standard deviation may be calculated as follows:

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$$s = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N - 1}}$$
$$= \sqrt{\frac{1.00 + .00 + 4.00 + ... + 1.00 + 9.00 + .00}{9 - 1}}$$
$$= \sqrt{\frac{20}{8}} = \sqrt{2.50}$$
$$= 1.58$$

From this value of the standard deviation, we conclude that the average deviation of the nine scores in this sample from the mean of 3.00 is equal to 1.58.

For the frequency rating variable, using the calculations in Table 4.4, the standard deviation is equal to the following:

$$s = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N - 1}}$$
$$= \sqrt{\frac{161.29 + 334.89 + 151.29 + ... + 127.69 + 1{,}069.29 + 53.29}{20 - 1}}$$
$$= \sqrt{\frac{3{,}940.20}{19}} = \sqrt{207.38}$$
$$= 14.40$$

Here, we can conclude that, in this sample of 20 participants, in rating the frequency of the word *sometimes,* the average difference between a participant's frequency rating and the sample mean of 33.30% was 14.40%.

Chapter 3 mentioned that descriptive statistics are often provided within the body or text of a paper. For the frequency rating example,

> The average frequency rating for the word *sometimes* was approximately one third the distance between *Never* and *Always,* representing a 33% frequency ($M = 33.30$, $SD = 14.40$).

In this example, the symbols M and SD represent the mean and standard deviation, respectively. Both measures of central tendency and variability are reported because they provide different pieces of information about the nature and shape of the distribution of scores for a variable.

Computational Formula for the Standard Deviation

The computational formula for the standard deviation (Formula 4-6) simply places the computational formula for the variance (Formula 4-4) within the square root symbol:

$$s = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N - 1}} \qquad (4\text{-}6)$$

The standard deviation for the simple example of nine scores and the frequency rating variable are calculated using Formula 4-6 in Table 4.7 (the values for the sum of scores squared [$(\Sigma X)^2$] and the sum of squared scores [ΣX^2] for the two examples are found in Tables 4.5 and 4.6). Similar to the variance, note the same value for the standard deviation is obtained regardless of whether the definitional or computational formula is used.

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TABLE 4.7 ⬢ CALCULATION OF THE STANDARD DEVIATION USING THE COMPUTATIONAL FORMULA, SIMPLE EXAMPLE AND FREQUENCY RATING VARIABLE

Simple Example	Frequency Rating
$s = \sqrt{\dfrac{\Sigma X^2 - \dfrac{(\Sigma X)^2}{N}}{N-1}}$	$s = \sqrt{\dfrac{\Sigma X^2 - \dfrac{(\Sigma X)^2}{N}}{N-1}}$
$= \sqrt{\dfrac{101 - \dfrac{729}{9}}{9-1}}$	$= \sqrt{\dfrac{26118 - \dfrac{443556}{20}}{20-1}}$
$= \sqrt{\dfrac{101 - \dfrac{81.00}{8}}{8}}$	$= \sqrt{\dfrac{26118 - 22177.80}{19}}$
$= \sqrt{\dfrac{20.00}{8}} = \sqrt{2.50}$	$= \sqrt{\dfrac{3940.20}{19}} = \sqrt{207.38}$
$= 1.58$	$= 14.40$

LEARNING CHECK 3:

Reviewing What You've Learned So Far

1. Review questions
   a. What is the relationship between the variance and the standard deviation?
   b. Is it possible to get a negative value for the variance or standard deviation? Why or why not?
2. Calculate the standard deviation (*s*) using the definitional and computational formula for each of the following data sets.
   a. 5, 3, 9, 2, 6
   b. 11, 19, 8, 10, 9, 7, 13
   c. 7, 9, 11, 12, 13, 14, 17, 21, 23
   d. 10, 8, 12, 8, 9, 7, 23, 6, 8, 11, 10, 6
   e. 11, 9, 13, 6, 14, 12, 7, 12, 10, 8, 15, 5, 12

"Eyeball Estimating" the Mean and Standard Deviation

We've now provided formulas for the most commonly used measures of central tendency and variability: the mean and standard deviation. As you'll be calculating these two statistics in virtually every chapter for the remainder of this book, it's important for you to perform these calculations correctly. However, we feel it's equally if not more important for you to develop the ability to look at a set of data and estimate what you believe are the mean and standard deviation before punching numbers into your calculator.

As an example of "eyeball estimation," imagine a political scientist asks 15 voters the question, "How many of the 50 U.S. states require someone to pass a background check before buying a gun from a private seller (a seller who is not a firearms dealer)?" Following are 15 hypothetical responses to this question:

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30 35 18 42 30 45 33 25 37 44 33 35 40 30 36

Before reaching for your pencil and calculator, we ask you to fill in the blanks in the following sentence: "*Most of the scores are between* _____ *and* _____." What percentage of the sample would you associate with the word *most?* We'll assume that "most" is more than "half" but less than "all"; for the sake of argument, let's assume "most" means about two thirds to three fourths (66%–75%) of the scores.

Imagine you say, "Most of the scores are between 30 and 40." In doing so, you've laid the foundation for estimating the mean and standard deviation. Your estimate of the mean could be located halfway between these two scores: Halfway between 30 and 40 is *35*. Your estimate of the standard deviation is the difference between the estimated mean and either of the two scores; the difference between your estimated mean of 35 and either 30 or 40 is *5* (35 − 5 = 30 and 35 + 5 = 40). Consequently, saying "Most of the scores are between 30 and 40" can be used to develop "eyeball estimates" of a mean equal to 35 and a standard deviation equal to 5.

In the above example, the calculated mean of the 15 scores is equal to 34.20 and the standard deviation is equal to 7.18. (For the sake of reference, as of mid-2017, only 18 states required background checks from private sellers [http://smartgunlaws.org/].) It is not at all important for your eyeball estimates to be 100% correct; instead, we've found these estimates help avoid the situation where students' calculations are very *in*correct. For example, in calculating the standard deviation (s), students sometimes forget the last step, which is to calculate the square root of the variance. In this example, this would result in reporting the standard deviation to be 51.60 rather than the correct 7.18. Given that the maximum value for this variable ("How many of the 50 U.S. states . . .") is 50, reporting a standard deviation of 51.60 is outside the realm of possibility. We've found "eyeball estimates" help students become more confident in assessing whether they've performed the calculations correctly as well as interpreting the calculated values of their statistics.

4.7 MEASURES OF VARIABILITY FOR POPULATIONS

The formulas for the variance and standard deviation described thus far are used for samples drawn from populations. But what if you were to collect data from the entire population? This section discusses two measures of variability for populations: the population variance and the population standard deviation.

The Population Variance (σ^2)

The variance for data collected from a population is called the **population variance** (σ^2) (σ is the lowercase Greek letter sigma), which is defined as the average squared deviation of a score from the population mean. The definitional formula for the population variance is provided in Formula 4-7:

$$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N} \qquad (4\text{-}7)$$

where X is a score for a variable, μ is the population mean, and N is the total number of scores.

The formula for the population variance differs from the formula for the variance s^2 (Formula 4-4) in two important ways. First, the mean in the formula is the population mean μ rather than the sample mean $\overline{X}$. Second, the sum of the squared deviations ($\Sigma(X-\mu)^2$) is divided by N rather than $N-1$; this is because you are no longer estimating the variability in the population from data collected from a sample but instead have collected data from the entire population.

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Imagine, for a moment, that the nine scores in the first column of Table 4.3 represent a population rather than a sample. Using the squared deviations for these scores calculated in the last column of Table 4.3, the population variance would be equal to the following:

$$\begin{aligned}\sigma^2 &= \frac{\Sigma(X-\mu)^2}{N} \\ &= \frac{1.00+.00+4.00+...+1.00+9.00+.00}{9} = \frac{20.00}{9} \\ &= 2.22\end{aligned}$$

From this calculation, you would conclude that, assuming these nine scores represent the entire population, the average squared deviation of a score from the population mean of 3.00 is 2.22.

The Population Standard Deviation (σ)

Calculating the population variance involves squaring the deviation of each score from the population mean μ. In order to obtain a measure of variability that represents the average deviation (rather than the average squared deviation) from the population mean, the square root of the population variance may be calculated. This is the **population standard deviation (σ)**, defined as the square root of the population variance. Mathematically, the population standard deviation is the square root of the average squared deviation from the population mean, and it represents the average deviation of a score from the population mean. Formula 4-8 provides the definitional formula for the population standard deviation.

$$\sigma = \sqrt{\frac{\Sigma(X-\mu)^2}{N}} \qquad (4\text{-}8)$$

Relying again on Table 4.3, the population standard deviation for the set of nine scores is calculated below.

$$\begin{aligned}\sigma &= \sqrt{\frac{\Sigma(X-\mu)^2}{N}} \\ &= \sqrt{\frac{1.00+.00+4.00+...+1.00+9.00+.00}{9}} \\ &= \sqrt{\frac{20}{9}} = \sqrt{2.22} \\ &= 1.49\end{aligned}$$

From these calculations, it may be concluded that the average deviation of the nine scores from the population mean of 3.00 is equal to 1.49.

Measures of Variability for Samples vs. Populations

The population variance and standard deviation are examples of parameters, which were introduced in Chapter 3. Parameters are numeric characteristics of populations and are distinguished from statistics, which are numeric characteristics of samples. Parameters refer to data from the entire population, whereas statistics refer to data collected from a sample of the population. This is summarized below for both a measure of central tendency (the mean) and a measure of variability (the standard deviation):

Target	Numeric Characteristic	Mean	Standard Deviation
Sample	Statistic	$\bar{X}$	s
Population	Parameter	μ	σ

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LEARNING CHECK 4:

Reviewing What You've Learned So Far

1. Review questions
   a. What is the main difference between the population standard deviation and the standard deviation?
   b. Under what conditions would you calculate the population standard deviation for a set of data rather than the standard deviation?
2. Calculate the population standard deviation (σ) using the definitional and computational formula for each of the following data sets:
   a. 2, 4, 5, 6, 8
   b. 8, 6, 4, 8, 3, 7
   c. 71, 84, 65, 78, 89, 72, 60, 85
   d. 3.0, 1.5, 3.5, 3.0, 2.0, 2.5, 4.0, 1.0, 2.0
   e. 10, 8, 12, 8, 9, 7, 23, 6, 8, 11, 10, 6

Because researchers rarely collect data from entire populations, statistics such as the sample mean and standard deviation are much more likely to be calculated than population parameters. Throughout this book, when you see the words *mean, variance,* or *standard deviation,* you should assume they refer to samples rather than populations. In fact, numeric values of parameters such as μ or σ are typically values believed or hypothesized to be true rather than values based on the actual collection of data.

4.8 MEASURES OF VARIABILITY FOR NOMINAL VARIABLES

The "frequency rating" variable used in this chapter consisted of numbers such as 46, 15, and 21; the higher the number, the higher the frequency rating of the word *sometimes.* In Chapter 1, we indicated that numeric variables whose values are equally spaced along a numeric continuum are measured at the interval or ratio levels of measurement (ratio variables differ from interval variables in that ratio variables possess a true zero point, meaning that the value of zero represents the complete absence of the variable). Chapter 3 discussed two measures of central tendency used with interval or ratio variables: the mean and the median. In this chapter, we've discussed measures of variability for interval or ratio variables: the range, variance, and standard deviation. However, variables can also be measured at the nominal level of measurement, whose values differ in category or type; examples of nominal variables are gender (male, female, etc.) and state (Missouri, Pennsylvania, etc.). In Chapter 3, we noted that the mode can be used to represent the central tendency of nominal variables; a measure of variability for nominal variables is discussed below.

The Index of Qualitative Variation

As a simple example of a nominal variable, imagine you're organizing a dinner and you ask 44 people whether they would eat a chicken entree or a vegetarian entree; 21 people say "either," 7 say "chicken," 13 say "vegetarian," and 3 say "neither." Following the guidelines discussed in Chapter 2, a frequency distribution table for these data is presented below:

Type of entree	*f*	%
Either	21	48%
Chicken	7	16%
Vegetarian	13	29%
Neither	3	7%
Total	44	100%

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Because "type of entree" is a nominal variable that consists of four distinct categories (either, chicken, vegetarian, neither), an appropriate measure of central tendency would be the mode, which is the score or value of a variable that appears most frequently in a set of data. Looking at the frequency distribution table, the mode type of entree is "either" because more people selected it ($f = 21$) than the other three possibilities. But looking at this frequency distribution table, we see that the four choices vary widely in their frequencies. What if we wanted to numerically represent the amount of variability in people's entree preferences?

The **index of qualitative variation (IQV)** is a measure of variability for nominal variables. The formula for the index of qualitative variation (IQV) is provided in Formula 4-9:

$$\text{IQV} = \frac{k(N^2 - \Sigma f^2)}{N^2(k-1)} \quad (4\text{-}9)$$

where k is the number of categories, N is the total sample size, and f is the frequency for each of the categories.

Using the information from the above frequency distribution table, the IQV for the type of entree example is calculated below:

$$\begin{aligned}
\text{IQV} &= \frac{k(N^2 - \Sigma f^2)}{N^2(k-1)} \\
&= \frac{4[(44)^2 - ((21)^2 + (7)^2 + (13)^2 + (3)^2)]}{(44)^2(4-1)} \\
&= \frac{4[1936 - (441 + 49 + 169 + 9)]}{1936(3)} \\
&= \frac{4(1936 - 668)}{5808} = \frac{4(1268)}{5808} = \frac{5072}{5808} \\
&= .87
\end{aligned}$$

Possible values of the IQV range from 0 to 1; as with any measure of variability, the smaller the value of the IQV, the less the amount of variability in the frequencies of the different values of the nominal variable. As an extreme example, if all 44 of the people in the sample had chosen chicken for their entree (implying no variability in people's entree preferences), the IQV would have been equal to .00. On the other hand, if an equal number of people had selected each of the four choices ($f = 11$ for all four choices), the IQV would have been equal to 1.00.

4.9 MEASURES OF VARIABILITY: DRAWING CONCLUSIONS

These early chapters of this book have discussed the importance of examining and drawing appropriate conclusions about three aspects of distributions: modality, symmetry, and variability. Measures of central tendency such as the mean, median, and mode provide a numerical description of the modality and, to a lesser degree, the symmetry of a variable by focusing on the center of the distribution and what scores have in common with each other. However, it is equally important to consider the variability in a distribution, which is the degree to which scores differ from the center of the distribution and from each other. Measures of variability such as the variance and standard deviation provide valuable information, for example, regarding the degree to which participants in a sample agree when asked the same survey question or respond in a similar manner to the same experimental manipulation.

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The beginning of this chapter introduced a research study designed to measure the degree to which people differ in their interpretation of words such as *sometimes, often,* and *seldom.* Based on the variability of the frequency ratings, what did these researchers conclude regarding the degree to which people agree in their perceptions of these words? Comparing the results of their study with those conducted in other countries, they concluded the following:

> There are subtle variations in how those sharing the same language describe the intermediate points . . . so it cannot be assumed that rating scales developed in one culture can be automatically used in another, even where there is a common language. (Skevington & Tucker, 1999, p. 59)

4.10 LOOKING AHEAD

Several critical issues have been identified and discussed in the first four chapters of the book. The first is the importance of examining data before conducting statistical analyses on the data. Appropriate and meaningful conclusions cannot be drawn from statistical analyses until the accuracy of the data has been confirmed and the distribution of scores has been understood. Second, there are many different types of distributions; you have seen distributions labeled as symmetric, skewed, peaked, flat, unimodal, and bimodal. Third, distributions can be described numerically using measures of central tendency and variability. The next chapter discusses yet another type of distribution: "normal distributions." Like the other distributions discussed so far, normal distributions can be described using descriptive statistics such as measures of central tendency and variability. What makes normal distributions unique, as we will discuss in greater detail going forward, is that they possess characteristics that enable them to be the basis of a second type of statistics, known as "inferential statistics," whose goal is to test hypotheses about populations based on the information from samples.

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4.11 Summary

Variability is a third aspect of distributions and refers to the amount of spread or scatter of scores in a distribution. One goal of a science such as psychology is to describe, understand, explain, and predict variability in the phenomena studied by researchers.

A ***measure of variability*** is a descriptive statistic of the amount of differences in a set of data for a variable. Common measures of variability are the range, the interquartile range, the variance, and the standard deviation.

The ***range*** is the mathematical difference between the lowest and highest scores in a set of data. The ***interquartile range*** is the range of the middle 50% of the scores for a variable, calculated by removing the highest and lowest 25% of the distribution. The ***variance*** (s^2) is the average squared deviation of a score from the mean. The ***standard deviation*** (s) is the square root of the variance, and it represents the average deviation of a score from the mean. Because the variance and standard deviation (unlike the range and interquartile range) are based on all of the scores in the distribution, they can be used in statistical analyses designed to test hypotheses about distributions.

The variance and the standard deviation may be calculated using a ***definitional formula,*** which is a formula based on the actual or literal definition of a concept, or a ***computational formula,*** defined as a formula not based on the definition of a concept but is designed to simplify mathematical calculations. Computational formulas are used because using the definitional formula in a large or complicated data set increases the chances of making computational errors.

The variance and standard deviation measure the variability in data collected from samples drawn from populations; the ***population variance*** (σ^2) (the average squared deviation of scores from the population mean) and the ***population standard deviation*** (σ) (the square root of the population variance) are calculated when data are collected from the entire population. Because data are rarely collected from entire populations, numeric values of parameters such as σ are typically based on beliefs and hypotheses rather than the actual collection of data.

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In addition to measures of variability for numeric variables measured at the interval or ratio levels of measurement, measures of variability also exist for categorical variables measured at the nominal level of measurement. The ***index of qualitative variation (IQV)*** is a measure of variability for nominal variables.

4.12 Important Terms

measure of variability (p. 88)
range (p. 89)
interquartile range (p. 92)
variance (s^2) (p. 94)
definitional formula (p. 96)
computational formula (p. 96)
biased estimate (p. 99)
unbiased estimate (p. 99)
standard deviation (s) (p. 100)
population variance (σ^2) (p. 103)
population standard deviation (σ) (p. 104)
index of qualitative variation (IQV) (p. 106)

4.13 Formulas Introduced in This Chapter

Range

$$\text{Range} = \text{highest score} - \text{lowest score} \tag{4-1}$$

Interquartile Range

$$\text{Interquartile range} = (N - \frac{N}{4})\text{th score} - (\frac{N}{4} + 1)\text{th score} \tag{4-2}$$

Variance (s^2) (Definitional Formula)

$$s^2 = \frac{\Sigma(X - \bar{X})^2}{N - 1} \tag{4-3}$$

Variance (s^2) (Computational Formula)

$$s^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N - 1} \tag{4-4}$$

Standard Deviation (s) (Definitional Formula)

$$s = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N - 1}} \tag{4-5}$$

Standard Deviation (s) (Computational Formula)

$$s = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N - 1}} \tag{4-6}$$

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Population Variance (σ^2) (Definitional Formula)

$$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N} \tag{4-7}$$

Population Standard Deviation (σ) (Definitional Formula)

$$\sigma = \sqrt{\frac{\Sigma(X-\mu)^2}{N}} \tag{4-8}$$

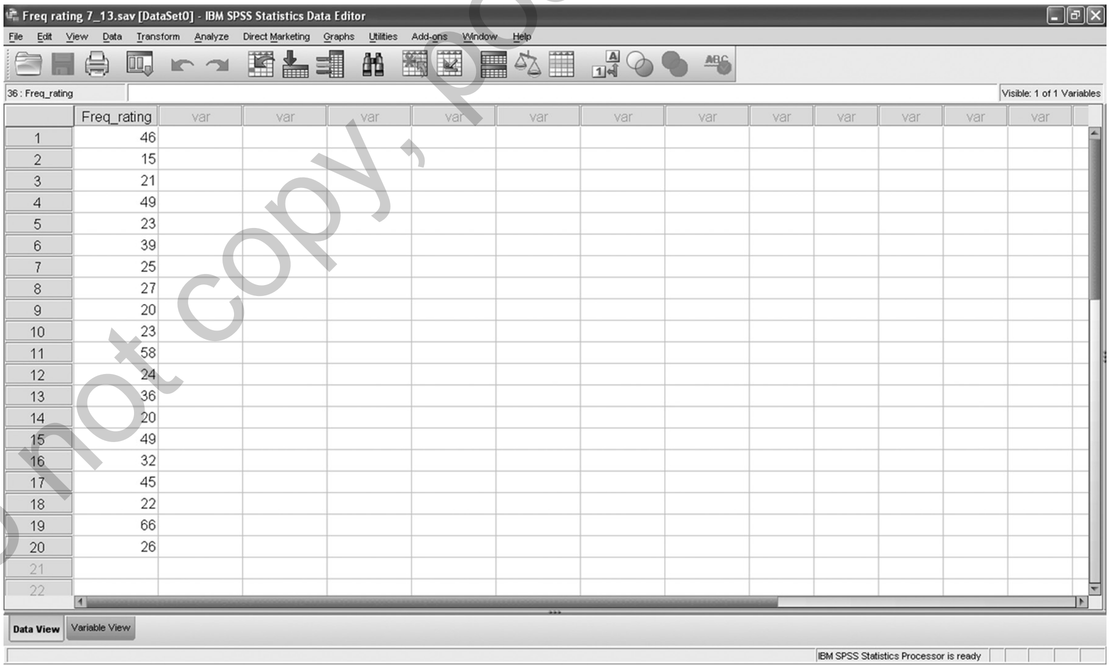
Index of Qualitative Variation (IQV)

$$\text{IQV} = \frac{k(N^2 - \Sigma f^2)}{N^2(k-1)} \tag{4-9}$$

4.14 Using SPSS

Calculating Measures of Central Tendency and Variability: The Frequency Rating Study (4.1)

1. Define variable (name, # decimal places, label for the variable) and enter data for the variable.



2. Select the descriptive statistics procedure within SPSS.

   How? (1) Click **Analyze** menu, (2) click **Descriptive Statistics,** and (3) click **Descriptives.**

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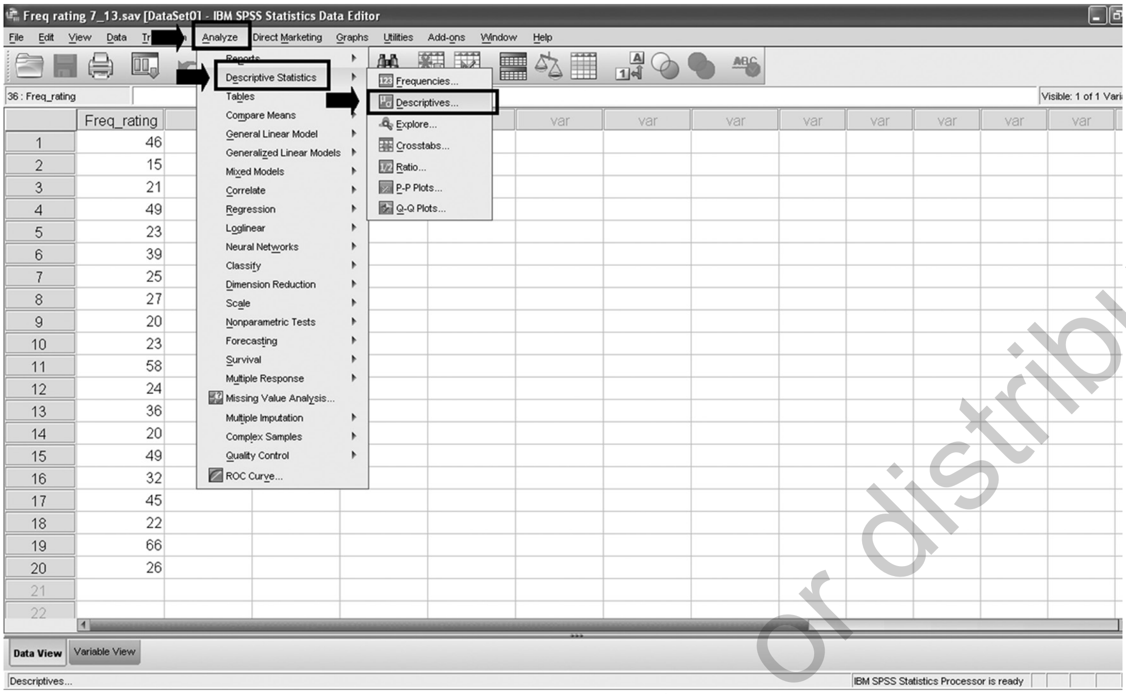
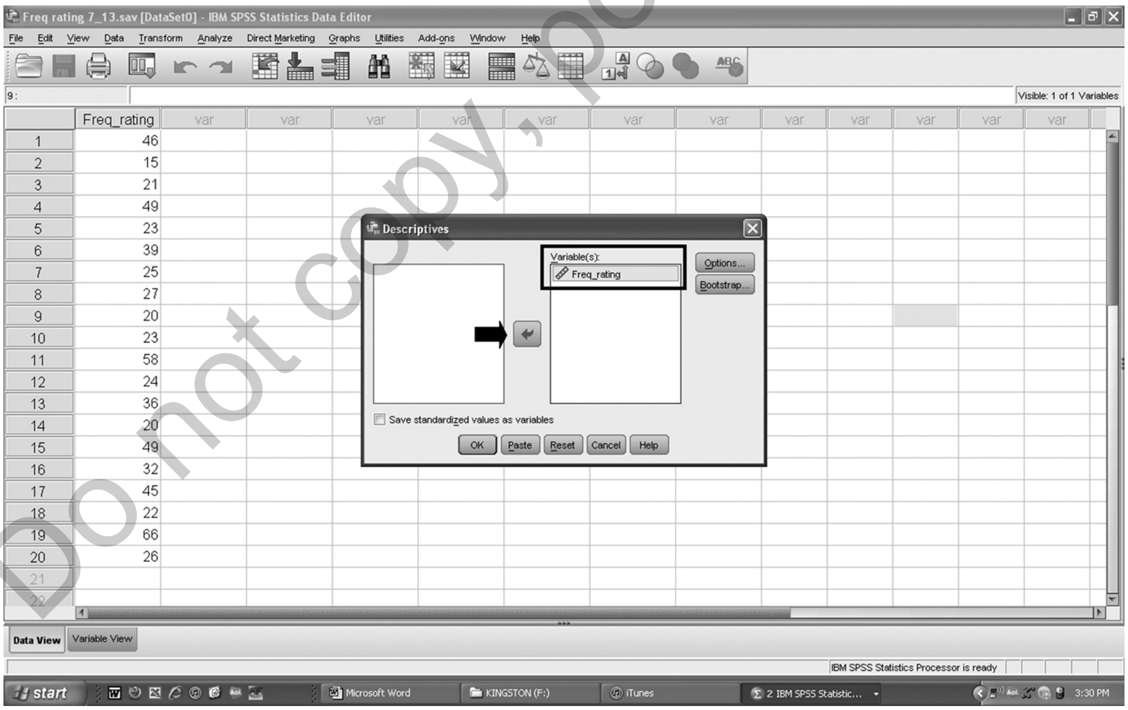


3. Select the variable to be analyzed.

How? (1) Click variable and →, and (2) click **OK**.



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4. Examine output.

N   Mean   Standard deviation

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
Freq_rating	20	15	66	33.30	14.401
Valid N (listwise)	20				

4.15 Exercises

1. Calculate the range for each of the following sets of data:
   a. 4, 6, 9
   b. 14, 17, 11, 19, 12
   c. 25, 22, 27, 30, 21, 26, 29
   d. 10, 8, 4, 16, 9, 7, 9, 13, 6, 11
   e. 73, 66, 91, 84, 69, 87, 62, 79, 82, 90
   f. 3.50, 4.21, 3.95, 2.27, 3.06, 4.58, 2.74, 3.89, 2.65, 2.03, 4.41, 3.76, 2.35
2. Calculate the range for each of the following sets of data:
   a. 9, 10, 13
   b. 5, 3, 9, 12
   c. 6, 2, 8, 12, 9, 5, 7
   d. 8, 4, 1, 6, 14, 9, 12, 5, 11, 7, 4
   e. 16.65, 12.98, 31.74, 18.80, 27.31, 29.92, 34.65, 23.68, 28.20, 20.77
3. Calculate the interquartile range for each of the following sets of data:
   a. 3, 6, 7, 12, 15, 17, 23, 28
   b. 8, 4, 1, 6, 13, 10, 12, 5
   c. 3, 8, 14, 11, 16, 7, 14, 15, 11, 9, 12, 6
   d. 15, 22, 17, 13, 31, 25, 22, 19, 26, 30, 27, 19, 23, 21, 27, 29
   e. 450, 560, 340, 510, 390, 670, 540, 420, 720, 480, 560, 510
   f. 29, 27, 26, 15, 28, 26, 32, 27, 26, 25, 30, 26, 27, 18, 23, 35
4. Calculate the interquartile range for each of the following sets of data:
   a. 1, 2, 2, 3, 4, 5, 7, 10
   b. 21, 8, 17, 7, 12, 19, 5, 12
   c. 6, 9, 5, 14, 3, 15, 19, 7, 13, 6, 8, 5
   d. 6.78, 7.81, 6.35, 9.65, 5.43, 8.62, 5.90, 7.13, 6.56, 3.27, 8.92, 4.49
   e. 78, 55, 82, 64, 93, 69, 71, 82, 59, 71, 76, 52, 89, 75, 81, 78

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5. For each of the following sets of data, (1) calculate the mean of the scores ($\overline{X}$), (2) calculate the deviation of each score from the mean ($X - \overline{X}$), and (3) check to see if the sum of the deviations equals zero ($\Sigma(X - \overline{X}) = 0$).
   a. 2, 4, 6
   b. 5, 6, 13
   c. 4, 7, 8, 9
   d. 5, 2, 7, 13, 11
   e. 3, 8, 14, 11, 16, 7, 14, 15, 11
6. For each of the following sets of data, (1) calculate the mean of the scores ($\overline{X}$), (2) calculate the deviation of each score from the mean ($X - \overline{X}$), and (3) check to see if the sum of the deviations equals zero ($\Sigma(X - \overline{X}) = 0$).
   a. 7, 5, 12
   b. 6, 7, 9, 12
   c. 6, 8, 4, 11, 3, 7
   d. 5, 7, 3, 1, 4, 8, 5, 2, 7, 9, 12, 4, 15
7. For each of the sets of data in Exercise 5, calculate the variance (s^2) using the definitional formula and the computational formula.
8. For each of the sets of data in Exercise 5, calculate the population variance (σ^2) using the definitional formula. Comparing your calculations for each data set with those done in Exercise 7, which is larger, the variance or the population variance? Why?
9. For each of the sets of data in Exercise 5, calculate the standard deviation (s) and the population standard deviation (σ). For each data set, which is larger, the standard deviation or the population standard deviation?
10. (This example was discussed in Chapters 2 and 3.) A friend of yours asks 20 people to rate a movie using a 1- to 5-star rating: the higher the number of stars, the higher the recommendation. Their ratings are listed below:

Person	# Stars	Person	# Stars	Person	# Stars
1	★★★	8	★★	15	★★★
2	★★★★★	9	★★★	16	★★
3	★★	10	★★★★★	17	★★★
4	★★★★	11	★	18	★★
5	★★★	12	★★★★	19	★★★★
6	★★★	13	★★★★	20	★★★
7	★★★★	14	★★★		

   a. Calculate the variance (s^2) and standard deviation (s) of the ratings using either the definitional or the computational formulas. (Note: From Chapter 3, you may have already calculated the number of scores (N), sum of scores squared ($(\Sigma X)^2$), the sum of squared scores (ΣX^2), and the mean ($\overline{X}$).)
   b. Based on your value of the standard deviation, what would you conclude regarding the degree to which these people agree or disagree about this movie?

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11. One study stated the research hypothesis, “Violence behavior in children may be reduced by teaching them conflict resolution skills” (DuRant et al., 1996). The variable “violence behavior” was measured by the number of fights in which each student was involved. Below is a frequency distribution table for the number of fights for 12 students.

# Fights	*f*	%
4	1	8%
3	2	17%
2	5	42%

# Fights	*f*	%
1	1	8%
0	3	25%
Total	12	100%

a. Calculate the variance (s^2) and standard deviation (s) of the number of fights using either the definitional or the computational formulas.

b. Based on your value of the standard deviation, what is the average difference between the number of fights a student got involved in and the mean?

12. (This example was introduced in Chapter 3.) The 10% myth study discussed in this chapter measured the beliefs of both psychology majors and non–psychology majors. The 39 non–psychology majors in this study provided the following values for the brain power variable:

a. Calculate the variance (s^2) and standard deviation (s) of the estimates using either the definitional or the computational formulas. (Note: From Chapter 3, you may have already calculated the number of scores (N), sum of scores squared ($(\Sigma X)^2$), the sum of squared scores (ΣX^2), and the mean ($\bar{X}$).)

Non–Psychology Major	Estimated Brain Power	Non–Psychology Major	Estimated Brain Power	Non–Psychology Major	Estimated Brain Power
1	15	14	10	27	15
2	45	15	20	28	25
3	10	16	50	29	10
4	40	17	25	30	20
5	5	18	45	31	10
6	10	19	40	32	5
7	50	20	25	33	15
8	45	21	10	34	10
9	10	22	5	35	5
10	10	23	15	36	60
11	35	24	5	37	10
12	5	25	40	38	5
13	15	26	30	39	15

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13. (This example was introduced in Chapter 2.) An instructor administers a 27-item quiz to her class of 25 students. Each student's score on the quiz is the number of items answered correctly. These scores are listed below:

    a. Calculate the variance and standard deviation of the ratings using the computational formulas.

Person	Quiz Score	Person	Quiz Score	Person	Quiz Score
1	22	10	17	19	20
2	15	11	18	20	21
3	11	12	14	21	17
4	19	13	10	22	18
5	12	14	6	23	15
6	21	15	16	24	24
7	22	16	20	25	22
8	19	17	21		
9	23	18	19		

14. On your own, create two distributions of 10 scores that have the same mean but differ in modality: one unimodal and one bimodal. Calculate the standard deviation of the two distributions. Is there greater variability in the unimodal distribution or the bimodal distribution?

15. On your own, (a) create two distributions of 10 scores that have the same mean but differ in their amount of variability, and (b) create two distributions of 10 scores that have different means but have the same amount of variability. (c) What does this imply about the information that is necessary to accurately describe a distribution of scores for a variable?

16. What if someone told you, "There is very little variability in the scores for my variable." From this statement, would you be more likely to describe the shape of the distribution as peaked or flat?

17. What if someone told you, "There is a great deal of variability in the scores for my variable." From this statement, can you tell whether the shape of the distribution is symmetrical or skewed? Why or why not? If not, what would you do to determine the symmetry of the distribution?

18. What if someone told you, "The mean age of the participants in this sample was 35.50 and the standard deviation was 2.53." Why would you interpret the sample differently if you had been told the standard deviation was 14.71?

19. An employment survey in 2009 was conducted to find the average starting salaries of people receiving doctorate (PhD) degrees in psychology (Michalski, Kohout, Wicherski, & Hart, 2011). The mean starting salary for psychologists in assistant professor positions in universities was approximately $60,000 with a standard deviation of $11,000. Clinical psychologists in their first year of practice also report a mean of approximately $60,000 but with a standard deviation of $16,000.

    a. Which type of psychologist has more variability in income?

    b. For which type of psychologist could you more accurately predict a salary for, and why?

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Answers to Learning Checks

Learning Check 1

2. a. Range = 6
   b. Range = 12
   c. Range = 19
   d. Range = .90
3. a. Interquartile range = 3
   b. Interquartile range = 8
   c. Interquartile range = 1
   d. Interquartile range = .30

Learning Check 2

2. a. $s^2 = 4.00$
   b. $s^2 = 4.00$
   c. $s^2 = 17.07$
   d. $s^2 = 105.79$
   e. $s^2 = 1.84$

Learning Check 3

2. a. $s = 2.74$
   b. $s = 4.04$
   c. $s = 5.33$
   d. $s = 4.55$
   e. $s = 3.12$

Learning Check 4

2. a. $\sigma = 2.00$
   b. $\sigma = 1.91$
   c. $\sigma = 9.58$
   d. $\sigma = .91$
   e. $\sigma = 4.36$

Answers to Odd-Numbered Exercises

1. a. Range = 5
   b. Range = 8
   c. Range = 9
   d. Range = 12

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e. Range = 29

f. Range = 2.55

3. a. Interquartile range = 10

b. Interquartile range = 5

c. Interquartile range = 6

d. Interquartile range = 8

e. Interquartile range = 110

f. Interquartile range = 2

5. a.

Score (X)	Mean ($\bar{X}$)	Mean ($X - \bar{X}$)
2	4.00	–2.00
4	4.00	.00
6	4.00	2.00
		$\sum(X - \bar{X}) = 0$

b.

Score (X)	Mean ($\bar{X}$)	Score – Mean ($X - \bar{X}$)
4	7.00	–3.00
7	7.00	.00
8	7.00	1.00
9	7.00	2.00
		$\sum(X - \bar{X}) = 0$

c.

Score (X)	Mean ($\bar{X}$)	Mean ($X - \bar{X}$)
5	8.00	–3.00
6	8.00	–2.00
13	8.00	5.00
		$\sum(X - \bar{X}) = 0$

d.

Score (X)	Mean ($\bar{X}$)	Score – Mean ($X - \bar{X}$)
3	11.00	–8.00
8	11.00	–3.00
14	11.00	3.00
11	11.00	.00
16	11.00	5.00

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Score (X)	Mean ($\bar{X}$)	Score – Mean ($X - \bar{X}$)
7	11.00	–4.00
14	11.00	3.00
15	11.00	4.00
11	11.00	.00
		$\sum(X - \bar{X}) = 0$

e.

Score (X)	Mean ($\bar{X}$)	Score – Mean ($X - \bar{X}$)
5	7.60	–2.60
2	7.60	–5.60
7	7.60	–.60
13	7.60	5.40
11	7.60	3.40
		$\sum(X - \bar{X}) = 0$

7. a. $s^2 = 4.00$
   b. $s^2 = 19.00$
   c. $s^2 = 4.67$
   d. $s^2 = 19.80$
   e. $s^2 = 18.50$

9. a. $s = 2.00, \sigma = 1.63$
   b. $s = 4.36, \sigma = 3.56$
   c. $s = 2.16, \sigma = 1.87$
   d. $s = 4.45, \sigma = 3.98$
   e. $s = 4.30, \sigma = 4.06$

   Note: In all data sets, the standard deviation (s) is larger than the population standard deviation (σ) because you are dividing by a smaller number ($N - 1$ rather than N).

11. a. $s^2 = 1.66, s = 1.29$
    b. The average difference between the number of fights of a student and the mean is 1.29 fights.

13. a. $s^2 = 19.89, s = 4.46$

15. a. Sample Data Set 1: 4, 4, 5, 5, 5, 5, 5, 6, 6, 6 = 5.10, $s = .74$
       Sample Data Set 2: 2, 2, 3, 4, 5, 5, 6, 6, 9, 9 = 5.10, $s = 2.51$
    b. Sample Data Set 3: 3, 3, 4, 4, 5, 5, 5, 6, 7, 7 = 4.90, $s = 1.45$
       Sample Data Set 4: 8, 8, 9, 9, 10, 10, 10, 11, 12, 12 = 9.90, $s = 1.45$
    c. Distributions with the same mean or the same standard deviation can be describing very different information; therefore, it is important to provide both measures of central tendency and variability to describe a set of data.

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17. There can be a great deal of variability in both symmetrical and skewed distributions. The simplest way to determine the symmetry of a distribution is to look at a figure of the distribution.

19. a. Clinical psychologists have more variability in their starting salaries.
    b. You could more accurately predict the income for a psychologist going into an assistant professor position because there is less variability in their starting salaries.

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